# Chapter 5: Key finding (Part 2)

**Outline**



## 5.1 Type of oscillation is determined by energy in system

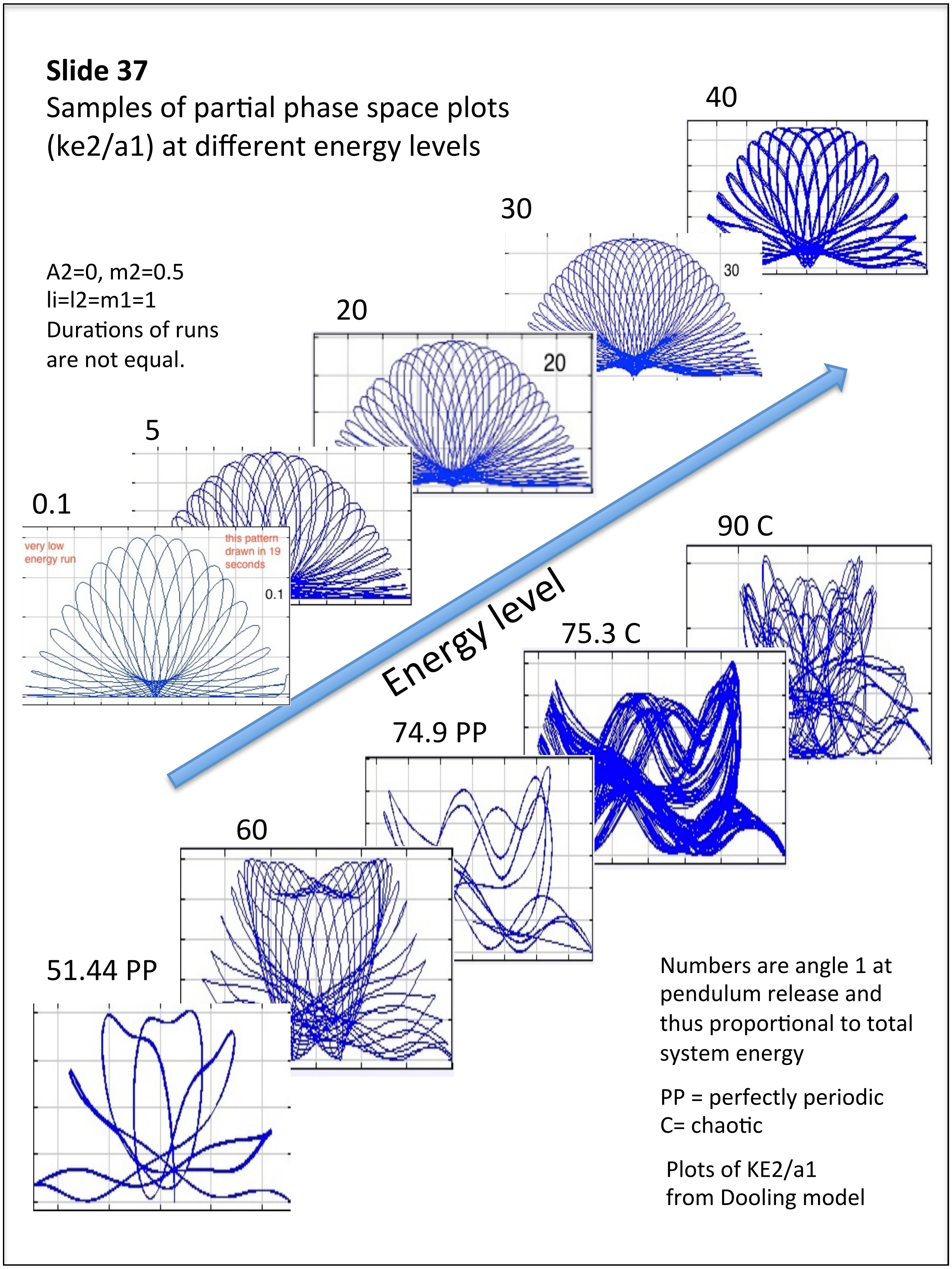
There are three general ways a system can oscillate: perfectly periodically, quasi-periodically, and chaotically. At least as far as the double pendulum is concerned the total amount of energy in the system determines which of these occurs.

The reason this matters is that each type of oscillation differs in terms of our ability to predict a systems future behavior based on historical records or simulation models. It also sets the conditions which anything dependent on it must cope with. For instance plants must cope with regular, predictable change in seasonal light and temperature. If we look at broad continent-wide seasonal averages they oscillate quasi-periodically. At local levels daily temperatures vary chaotically.

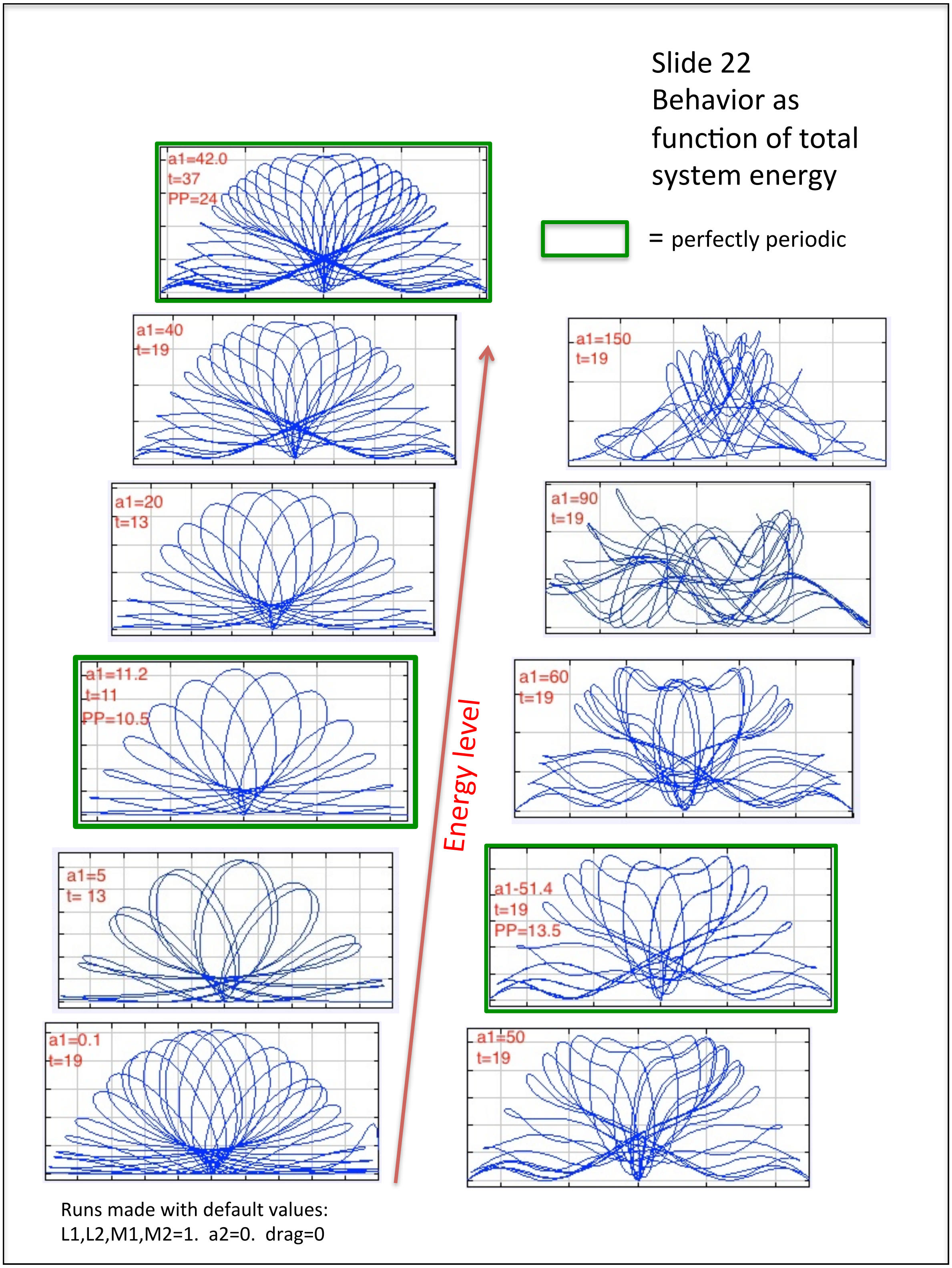
The behavior of systems other than the double pendulum depends on total energy in the system. Clearly that's true in laboratory Raleigh-Benard cells, which contain fluids heated from below. And it clearly applies to the Lorenz waterwheel.

Here I present an analysis of double pendulum behavior to support what I suspect is a generalization applying to all dynamic systems; namely that the level of energy within a system dictates its behavior. As mentioned above the physical design of the system also dictates behavior but for any given physical design behavior depends mainly on the energy level. (Arm positions at the beginning of a double pendulum run also dictate behavior to some extent)

Slides 37 shows the partial phase space plots produced by a series of simulation runs made at different release angles and with m2 set to 0.5. "a1" was the independent variable and ranged from 0.1 to 90 degrees in this series. At this point simply note that the patterns of behavior was different at different energy levels. Note also the oscillation in some runs was perfectly periodic (PP) and in other chaotic (C).




Slide 22 presents the same information for a second series of runs made with all the same initial conditions in terms except m2 was set to 1.0.




When friction is introduced it drains off energy and the system transitions through a range of behaviors. Slide 5 illustrates this. The first 25 seconds were clearly chotic as boht the top images show. At some point it became quasi-periodic and stayed that way in this particular run. Note how the intensity of the oscillations died down.

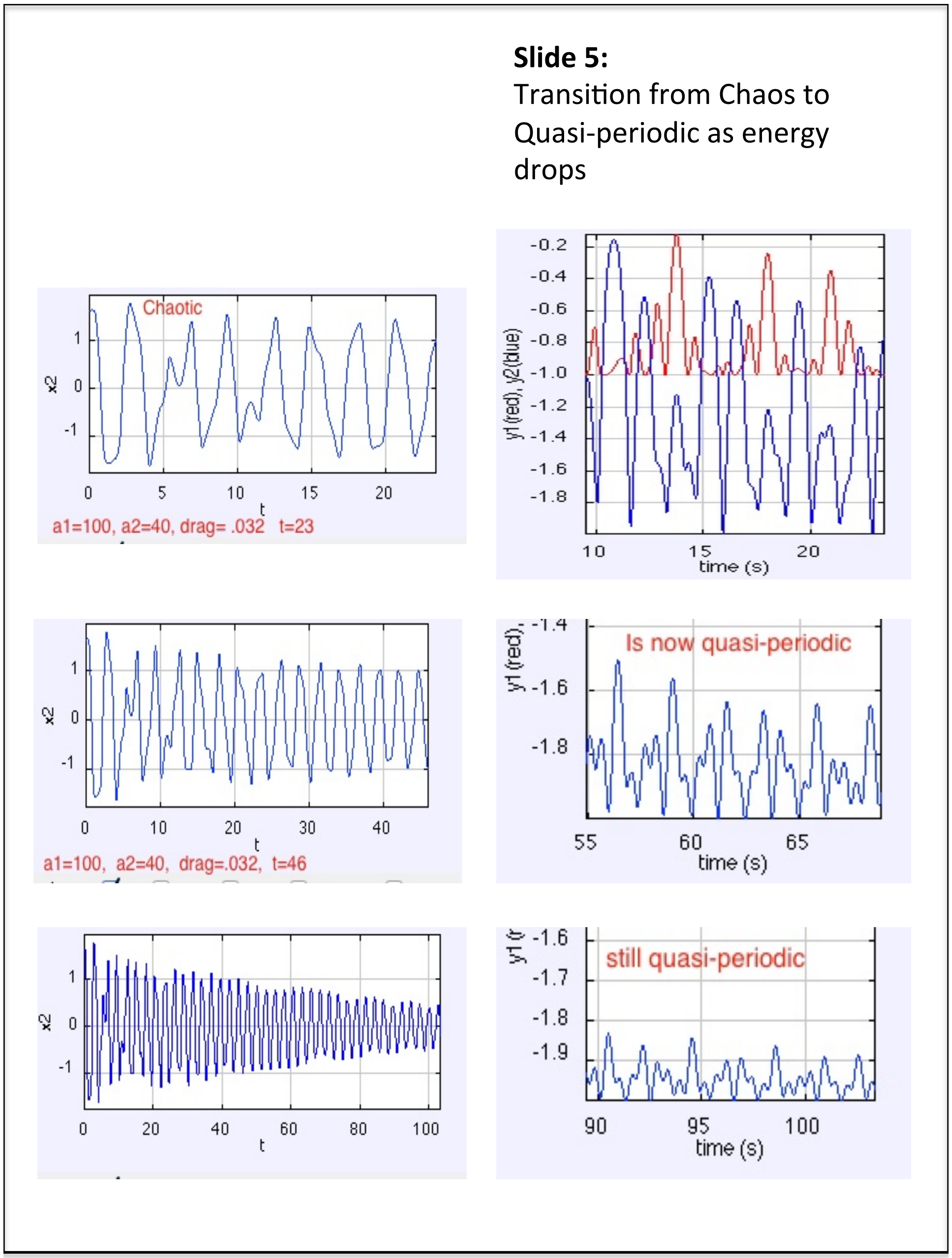
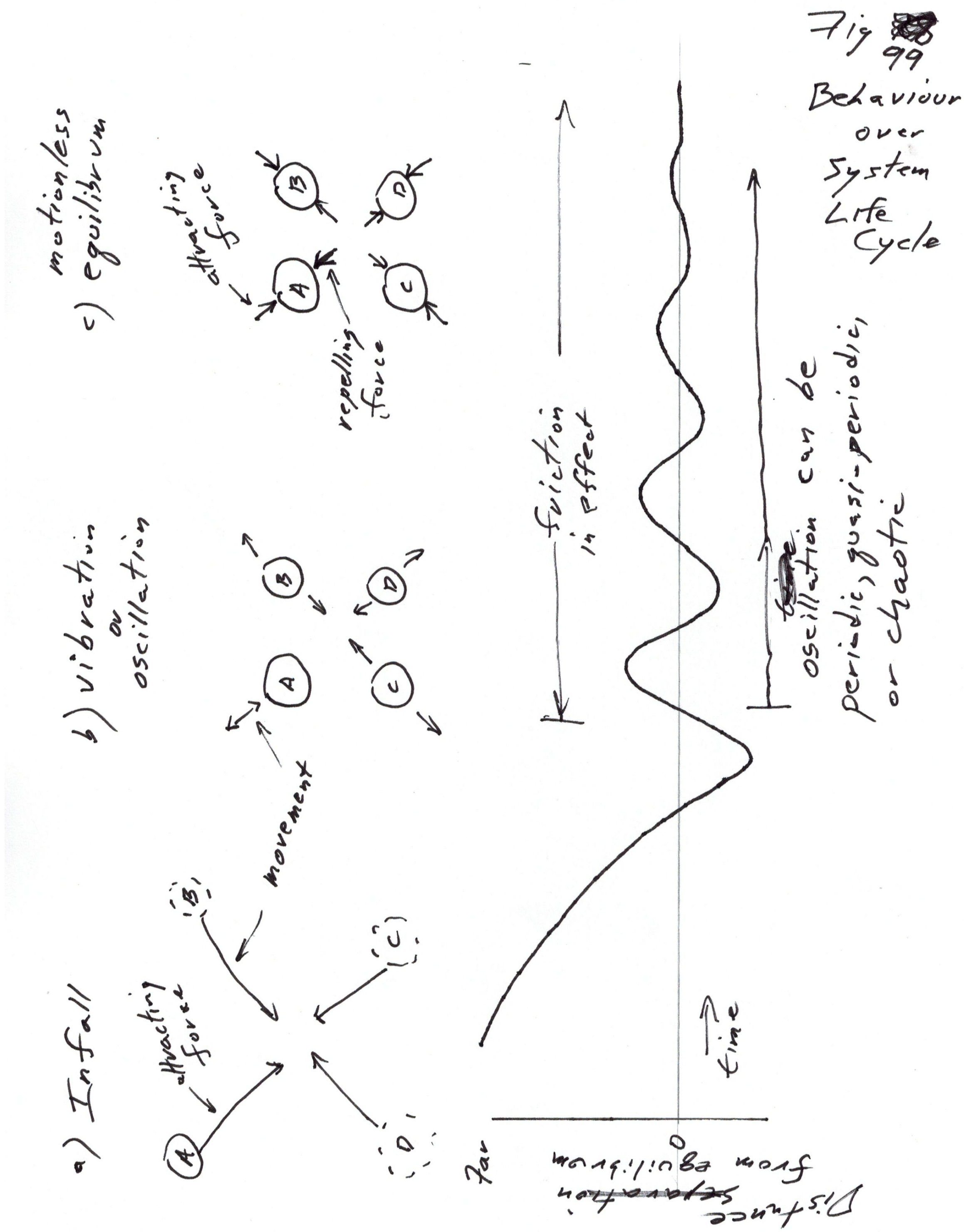
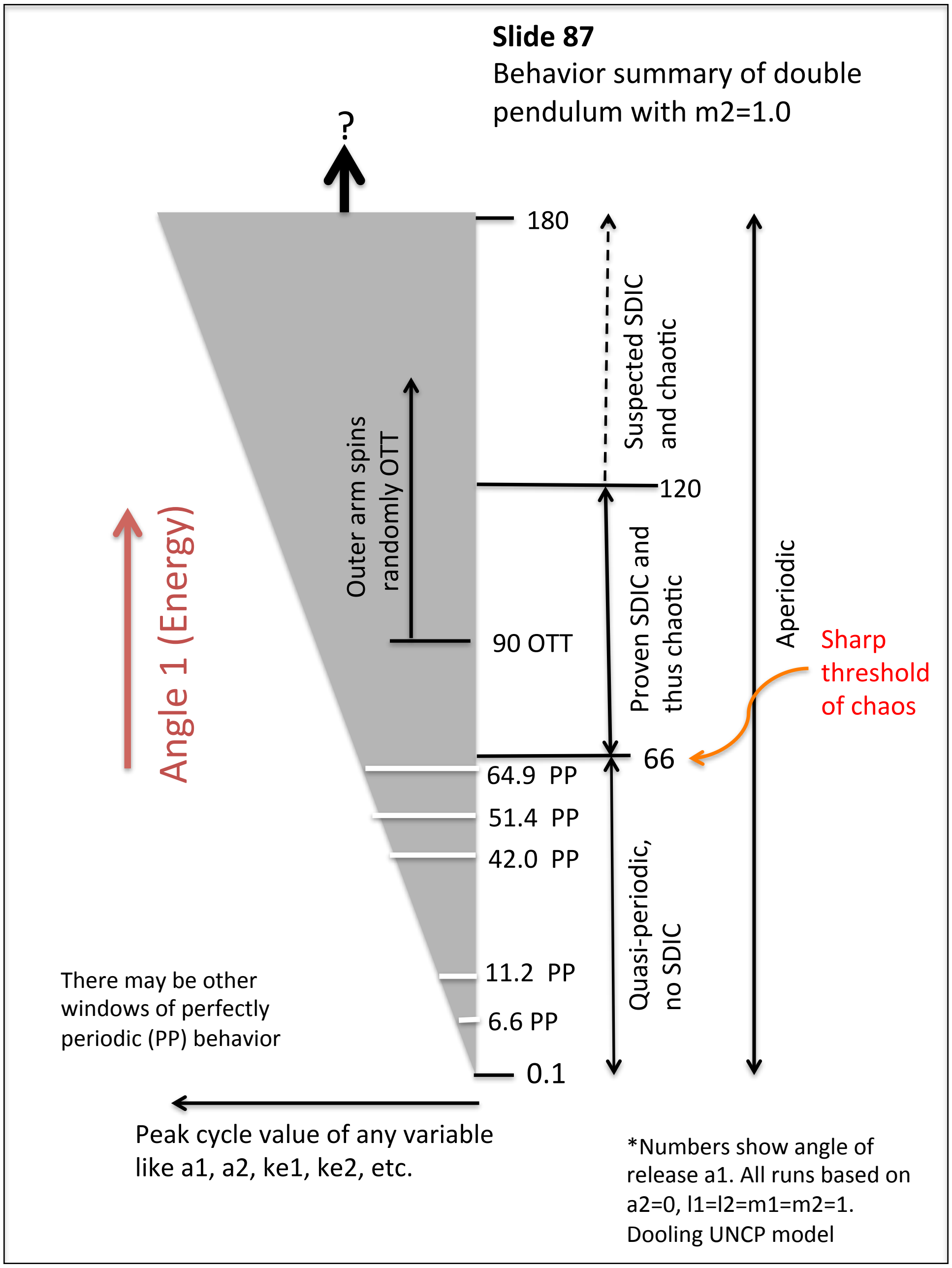



Figure 99 shows that systems go through a life cycle of sorts. It starts with the individual parts far apart but attracted toward each other by some force so they "fall" toward a central location. In terms of the universe the big bang separated all matter and its essentially been falling back together ever since. When the parts get close enough some repelling force will slow, stop and reverse the in-fall. And it will leave the parts oscillating about an equilibrium position. Friction or its equivalent will dampen the oscillation until the parts come to rest in an equilibrium configuration. This is of course a gross simplification. Still, it seems to apply to galaxies, solar systems, and molecules.



Slide 87 summarizes the behavior of the double pendulum as a function of total system energy in more detail. Its based on quite a few simulation runs I conducted. A much more detailed description of this figure and the runs behind it are presented in Chapter 7. Note that the specific numbers on this chart apply only to a double pendulum with the bob masses, arm lengths, and arm release angles specified. Change any of these physical constrains and the same general diagram will probably apply, albeit with different numbers. Recall that the total energy is determined by how high the bobs are lifted before they are released to start a run. I determine that by setting angle 1 (a1). I explored the territory by making runs with successively higher values of a1.




The general message conveyed is that the total amount of energy in the double pendulum system determines how it behaves.   Key points are:

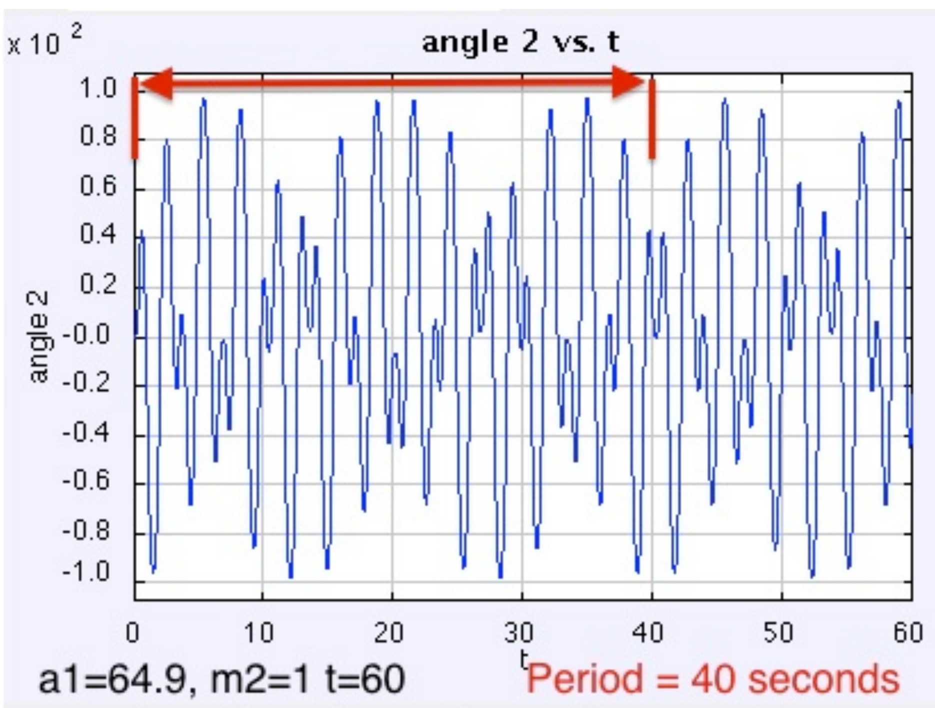
> a) With the exception of very narrow windows where the system oscillated in a perfectly periodic manner its oscillation was always aperiodic as shown by the arrow at right. This means there was no sequence of waves that ever repeated perfectly time after time.  If there had been the duration of that sequence would be called a period.

b) At lower energy levels (angle 1 under 66 degrees) this particular system was generally what's called quasi-periodic meaning that there were sequences of waves that looked <u>about</u> the same time after time but the wave heights weren't exactly the same height each time.

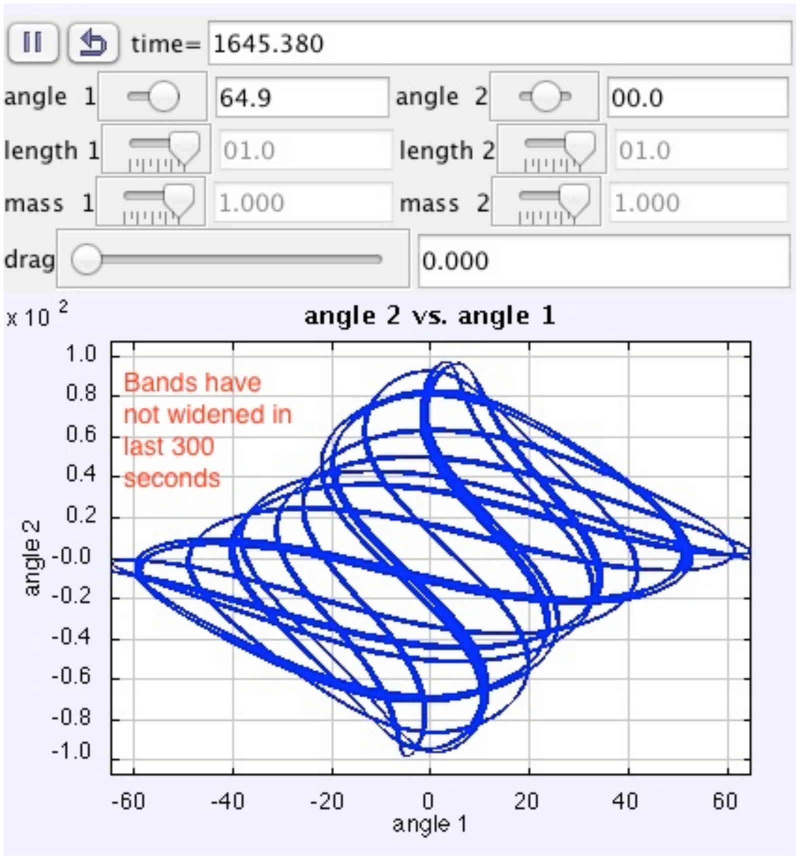
c) At 66 degrees there was a sharp break in behavior between quasi-periodic and chaotic because at all higher energies the system had a condition called sensitive dependence on initial conditions or SDIC. Most experts agree that a system must have SDIC to be "officially" labeled as chaotic.

d) The final major point is that at a1=90 there was another sharp break in behavior. Below that energy level the outer arm could never swing over the top as opposed to falling back like a backyard swing does. Above that energy level it could do that on random occasions. This qualitative difference in behavior produces what I call "dramatic events". Going over the top is a dramatic event.

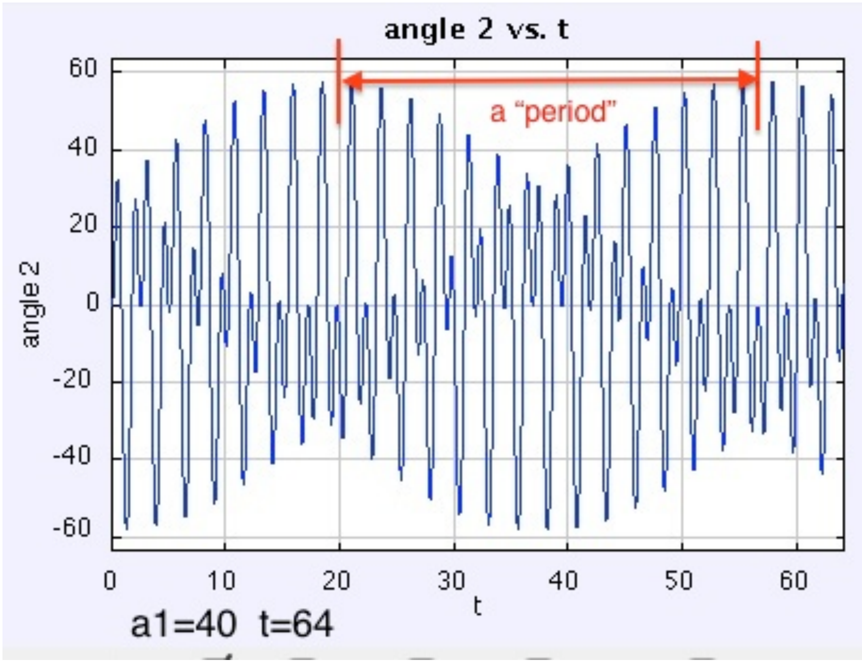
Perfectly periodic operation produced the following two plots. In the first the wave



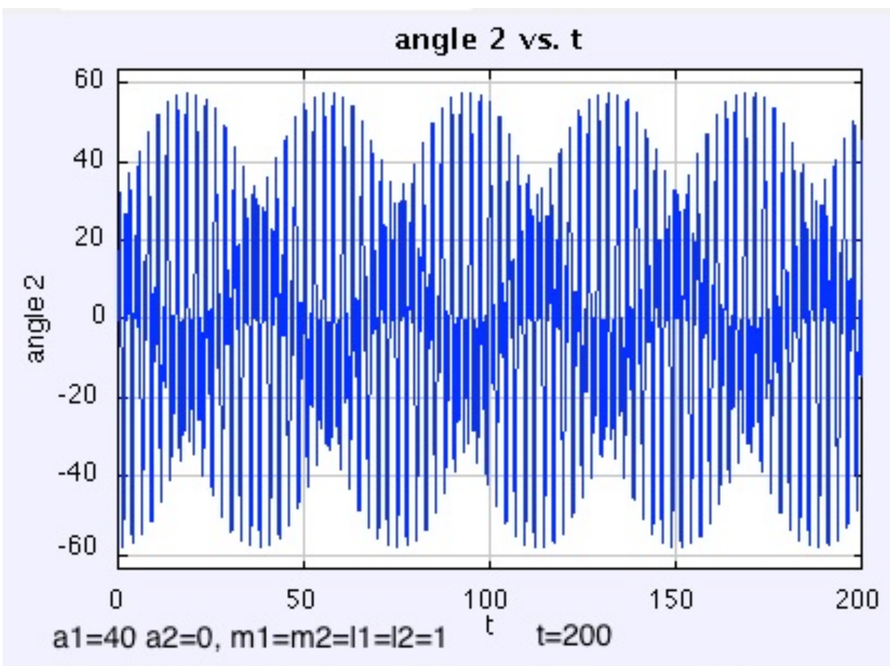

In the plot below the trace redrew almost exactly the same pattern many times over a very long time. Chapter 8.3 explains why it's not possible to visually determine whether this is truly perfectly periodic behavior or a version of quasi-periodic that's very close to it. It came down to judgment call on my part. I chose to call this perfectly periodic ,and its my best example of it.



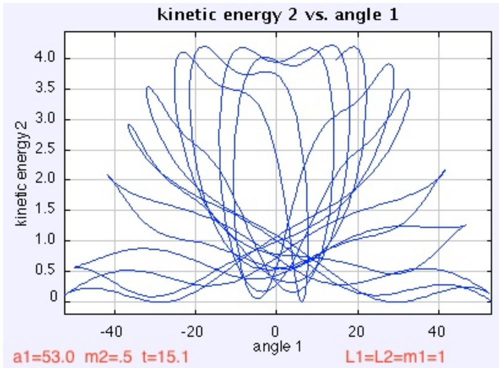

Quasi-periodic operation produced the following plots: In the first screenshot the waveform between t=0 and t=37 seems to begin repeating, and doing so every 37 seconds. But we can't tell by eye whether all the waves or peaks are exactly the same height.



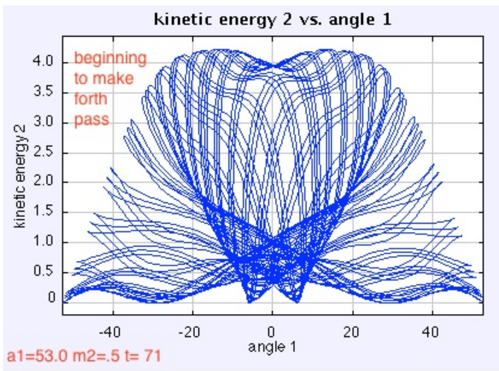

This is a longer-term view.



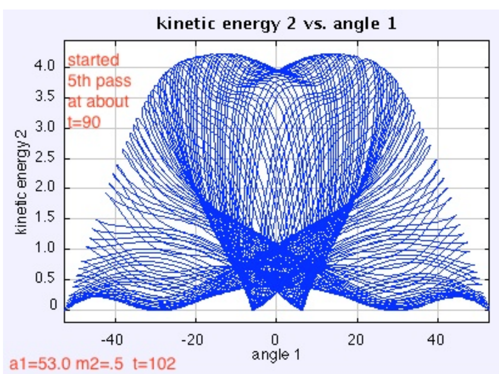

Its much easier to see if the waveform repeats exactly by plotting one variable against another. If so each new pattern will lie exactly atop the prior ones thus staying on a single sharp line. But here we see that each pass is somewhat offset from the prior one meaning this run isn't perfectly periodic. If run long enough the trace will fill the entire envelope. The amount of energy in the system determines how large this envelope is.




**Slide 32**
Quasi-periodic operation eventually fills envelope

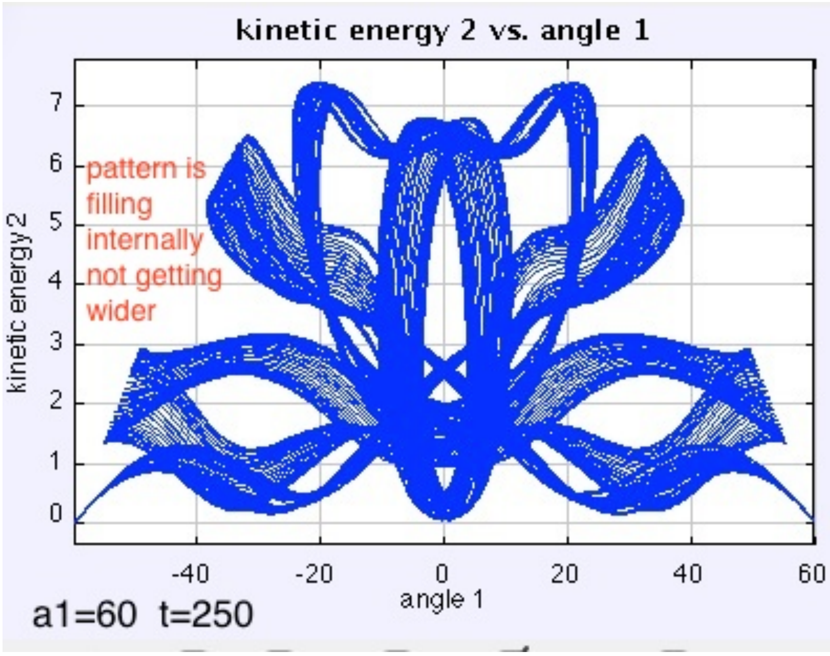



This is similar to chaos where each variable eventually peaks at all possible values within the range.




Note that values drift faster the further run is from being perfectly periodic. I.E.: pattern to pattern drift is greater.

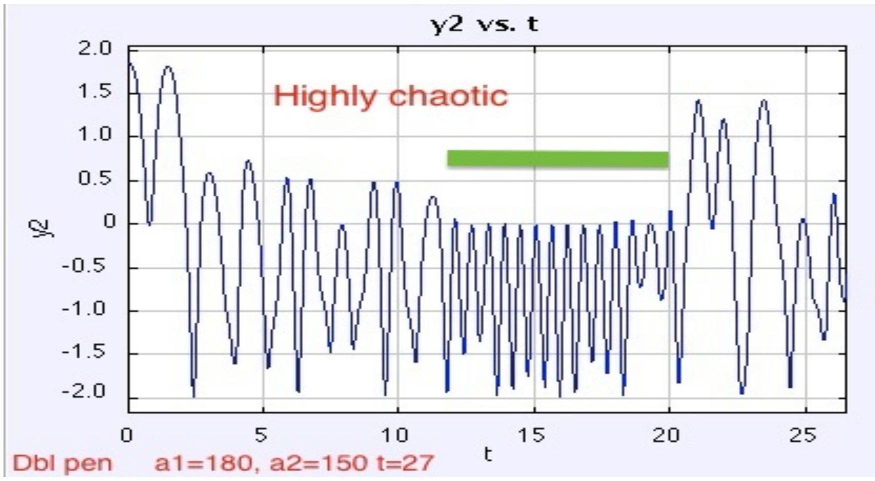
Quasi-periodic behavior can also produce a plot like the following, where the trace stays within a band.

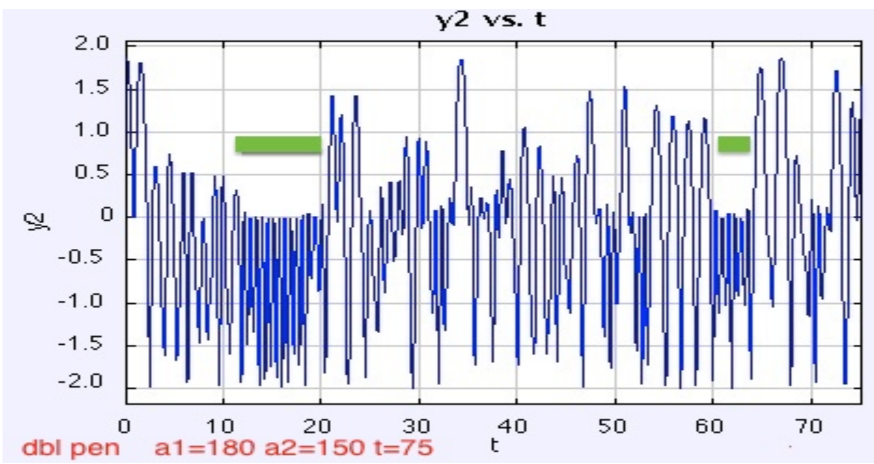



Slide 26 shows highly chaotic waveforms. They are characterized by random length periods where a given variable oscillates gently and then spikes to a much higher value. Green bars highlight the calm periods.

**Slide 26: Highly chaotic waveform in double pendulum**

Note random occurrence of spike values




Green bars highlight the longer calm periods
Y2 is height of outer bob re main pivot point thus proportional to its potential energy or PE. At -2 its PE is zero

Slide 58 compares the partial phase space plots for two runs made almost at the same energy level. The pair with a1=105 degrees is chaotic but just over the threshold. It visually apparent that this pattern shows no sign of repeating. It simply looks messy. The pair with a1=100 is just below the chaos threshold so is called quasi-periodic. Nevertheless its fairly messy as well. These runs have shown that there is a range of "quasiness" which spans from being almost perfectly periodic at one extreme to being almost chaotic at the other.

I haven't found plots like these for other systems like the Lorenz waterwheel or Lorenz equations. Although Stogatz has this to say about them:

> “What happens if we change the parameters? Its like a walk through the jungle –one can find exotic limit cycles tied in knots, pairs of limit cycles linked to each other, intermittent chaos, noisy periodicy, as well as strange attractors” (Ca2 p’330)

I should note that Strogatz and other experts do not often link behavior directly to energy. Often they describe how behavior changes with a “driving force” or a variable called the Rayleigh number. Still when the water flow to the Lorenz waterwheel is increased it must increase the level of energy, and it clearly does change behavior. See the Strogatz demo at:
https://www.youtube.com/watch?v=7iNCfNBEJHo

Finally, the behavior of fluid convection currents in a container heated from below changes depending on how much heat is applied. The topic of Rayleigh-Benard cells or convection if where to find details about that. There are also uTube videos on the subject.

I think these examples adequately support my conclusion that system behavior depends on energy. If and how something analogous applies to economic and other societal systems is TBD.

## 5.2 Thresholds to chaos and dramatic events are sharp

I can’t say this is a generalization, but its clearly true for the double pendulum. What’s meant here is that a very small increase in energy will cause the double pendulum to switch from non-chaotic operation to chaotic operation. A separate small increase in energy will enable the outer arm to occasionally go over the top as opposed to always falling back. It can make a practical difference which side of this threshold the system is on. Whether it is chaotic or not affects our ability to predict its future state. Whether it goes over the top or not is of course of no practical consequence but a change in behavior like that might hypothetically affect which way some ocean current circulates. I’ll expand on both these ideas later.

Its easy to find the threshold between non-periodic operation and chaotic operation by testing for the presence of sensitive dependence on initial conditions (SDIC). There are two other criteria in the most common definitions of chaos, but that seems to be the one most relevant here. I believe the double pendulum meets the other two.

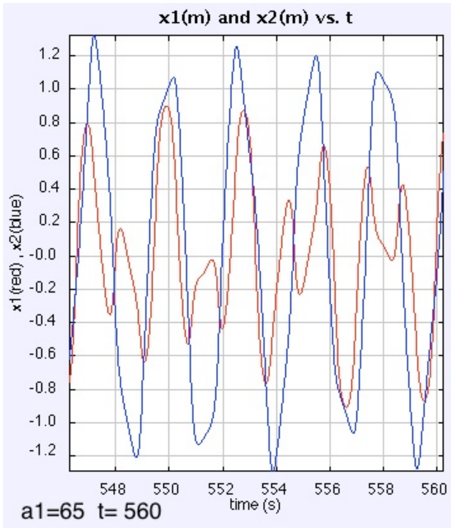
**Double pendulum:** One tests for SDIC by making two computer runs with almost the same initial conditions and comparing their waveforms. I use slightly different values of a1 which correspond to slightly different energy levels.  The Dooling model can only simulate one double pendulum at a time so one must compare the waveforms from two separate runs.  The ideal model would simulate two pendulums and plot both waveforms at the same time for easy comparison.

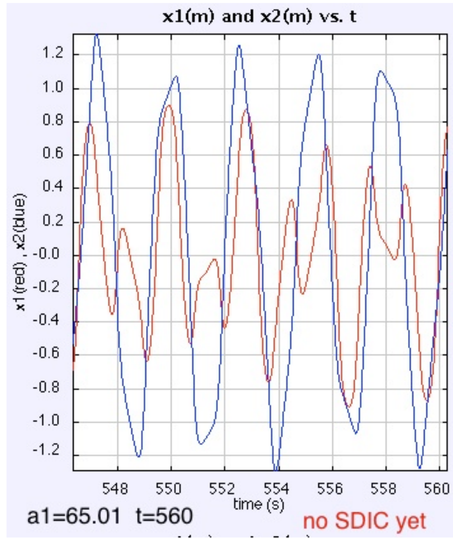
A pair of runs was made at a variety of ever-higher energy levels until one became SDIC and thus chaotic.

Slide 106 show the results of a test for SDIC with a1 =65 degrees..  Even after two very long 560-second runs the right and left waveforms appear identical.  The small 0.01 degree difference in initial conditions ( a1 and thus energy level) made no difference so there was no SDIC at this energy level.   It also turned out that this run was perfectly periodic.  This happened in another set of runs as well raising the possibility that the double pendulum becomes perfectly periodic just prion to becoming chaotic.

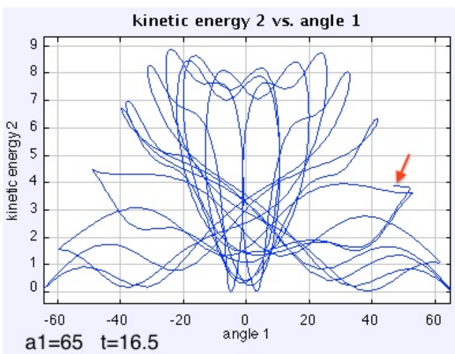
**Slide 106**
Development of SDIC
at angle 65

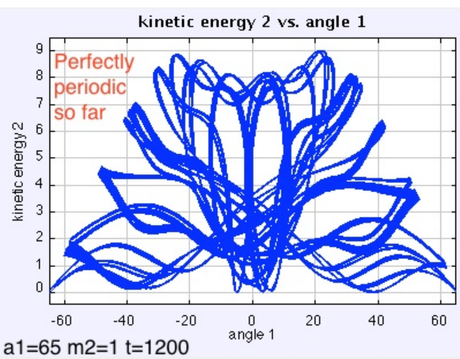
Stayed perfectly periodic with no SDIC so far, thus hasn't become chaotic.




Period was about 14 seconds



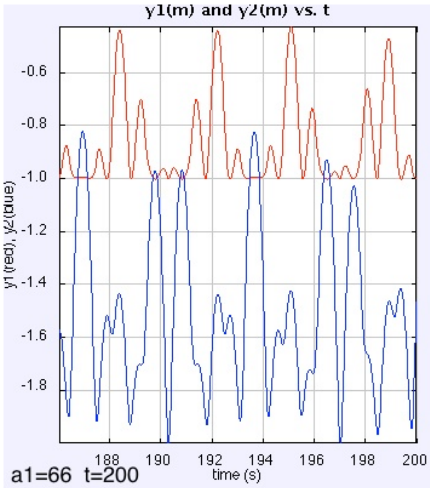

A2=0 m1=m2=l1=l2=1

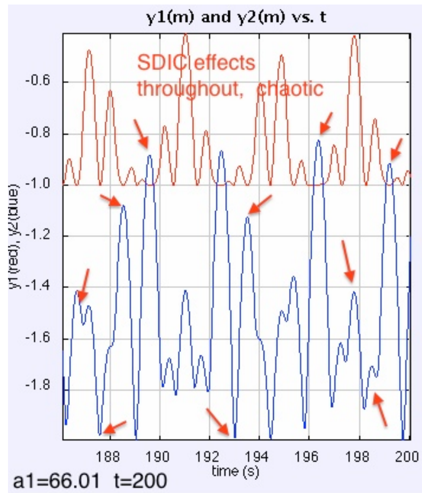
There was no change in width of bands from t=421 to t=1200

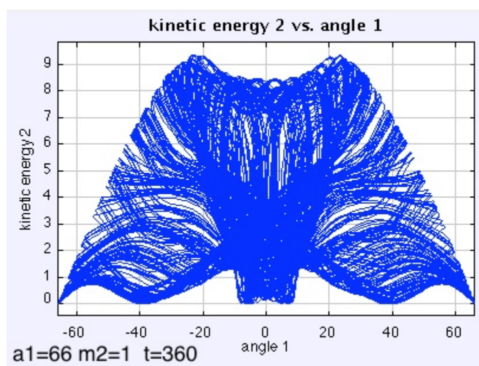
Slide 105 shows the system was SDIC when a1 was raised just one degree to 66 degrees. Because it was SDIC it was chaotic. The threshold was sharp because an increase of only 1 degree –representing a small change in energy level- was sufficient to shove it over.

Run had significant SDIC and thus was chaotic.

**Slide 105**
Development of SDIC at angle 66



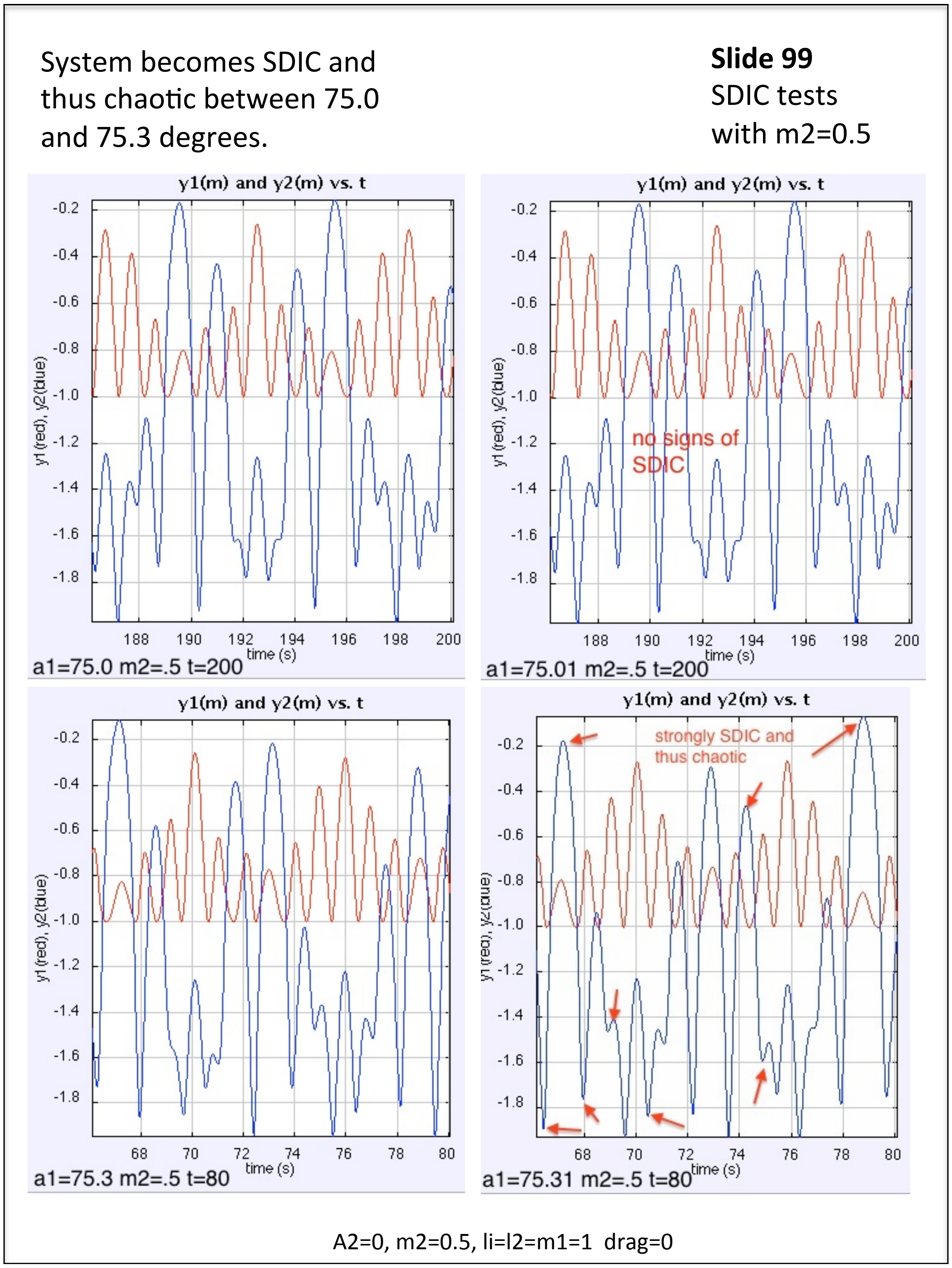



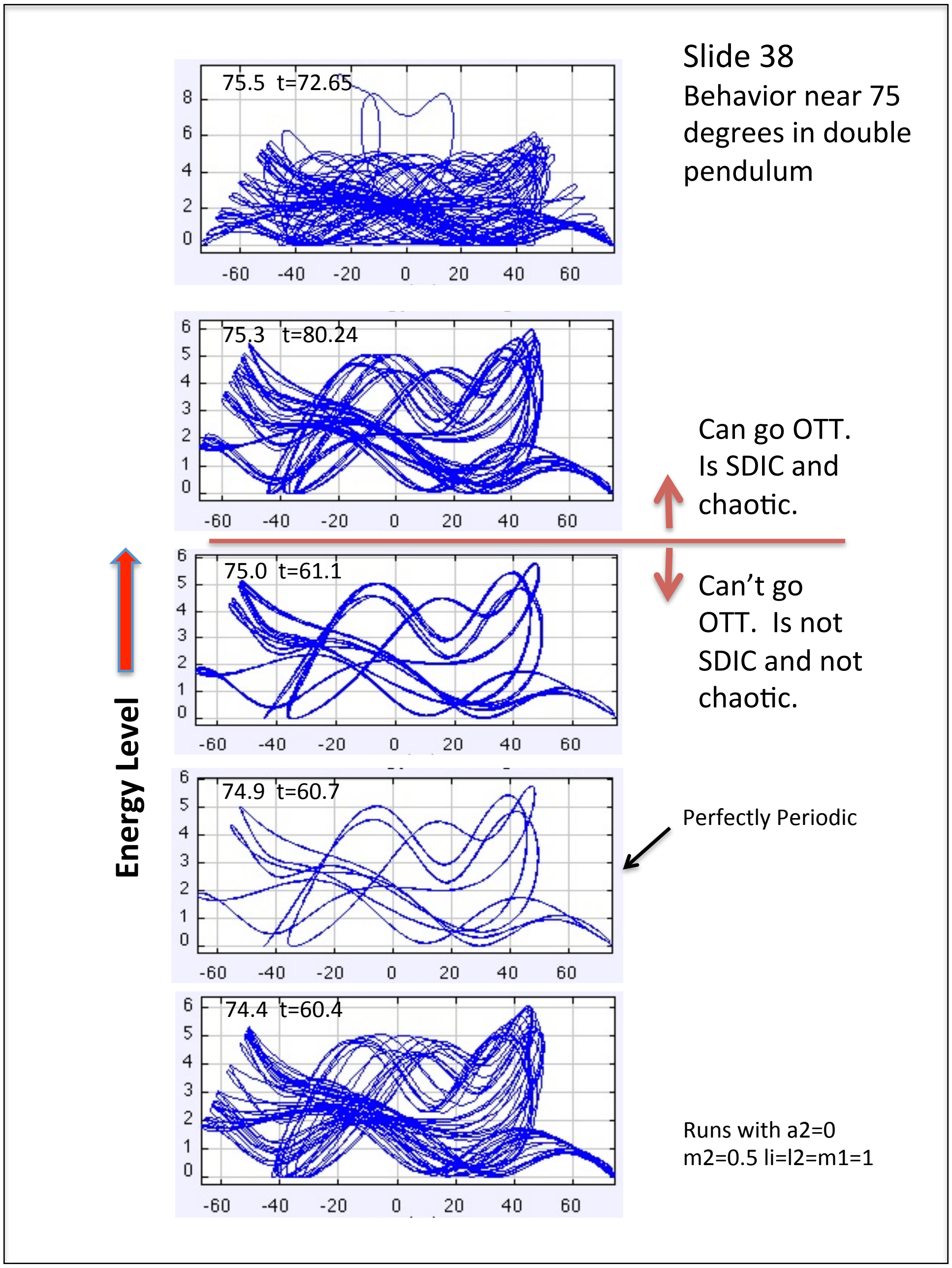

A2=0 m1=m2=l1=l2=1

Slide 99 shows the results of testing a different configuration in that the mass of the outer bob was doubled. One set of runs was slightly below the chaos threshold the other slightly above. The precise threshold is somewhere in between. I judge that a sharp threshold since angle one only differed by 0.3 degrees.

To interpret these plots look first at the top two. One was made with a1 at 75.0 degrees and the other with a very slightly different value of 75.01 degrees. SDIC would have amplified the effect of this small difference making the waveforms diverge over time, but here they look identical as best the eye can tell. In short I judged that SDIC was not present at this energy level. In contrast the waveforms in the two lower runs did become different in a relatively short time. The red arrows highlight some of those differences. Note here we are talking about differences in the wave height (value of some parameter) at some specific time. That what we can't predict if a system is chaotic. The general nature of the waveforms are similar. Both oscillate up and down about the same amount, and irregularly.



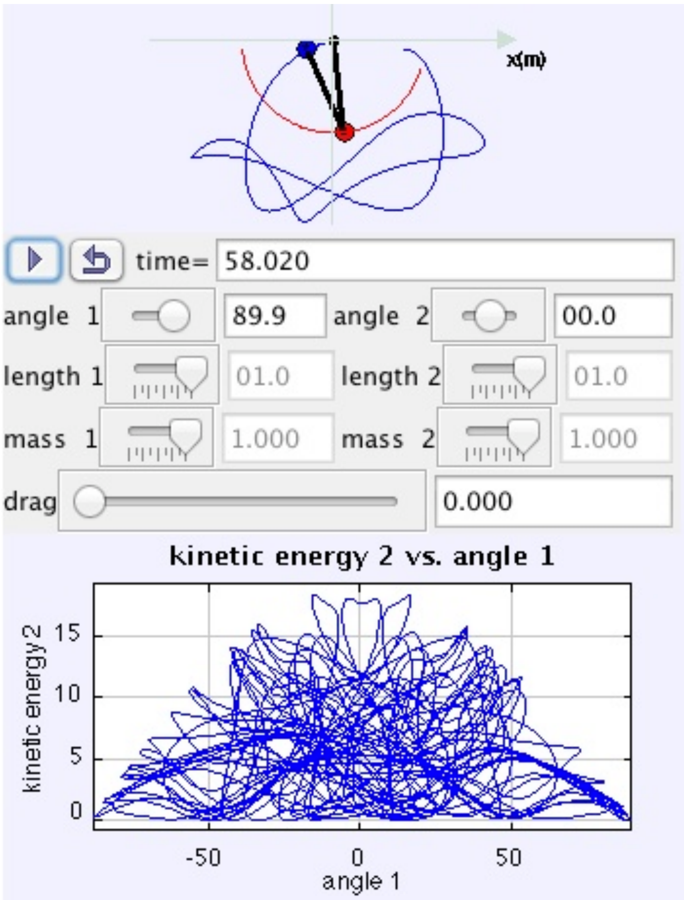

Slide 38 also shows how sharp this boundary is. At 75 degrees the system was not chaotic. At 75.3 degrees it was.



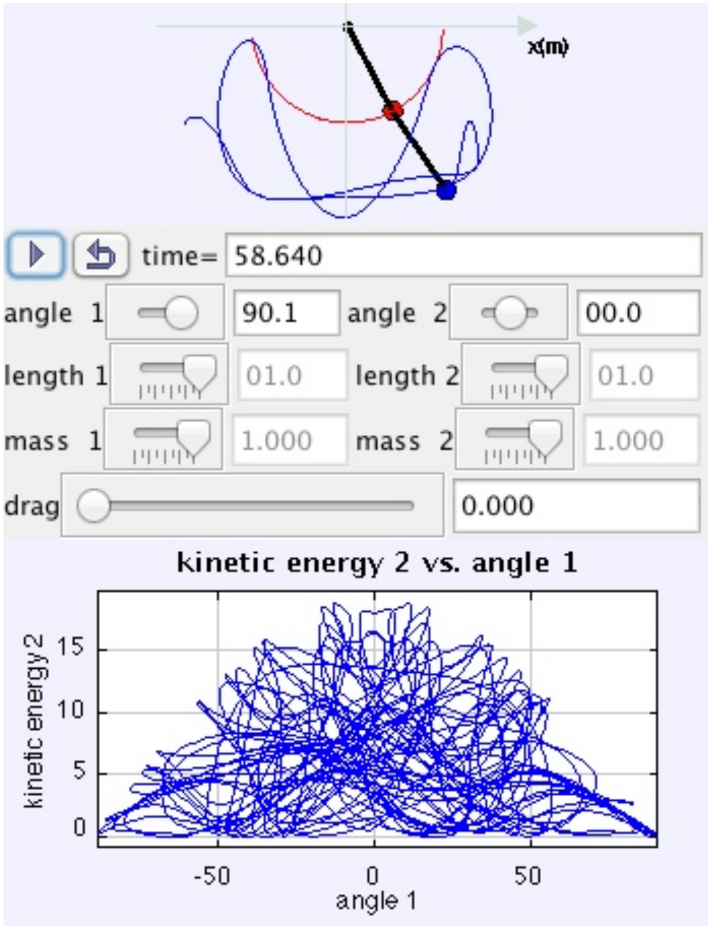

These runs and many more associated with it are detailed in Chapter 7.

**Threshold to dramatic events:** Below a certain energy level the outer bob of the double pendulum will occasionally swing up near the top, pause for a moment and then reverse and fall back because it doesn't have quite enough energy to go over. Its like rolling a ball up a round-top hill but not quite fast enough. However there is a point where it has just enough speed and energy to go over the top (OTT). There is a very small difference in the level of energy where it falls back and where it occasionally goes over. This is a fairly obvious point. I think the Lorenz waterwheel behaves that same way. At some sharp energy threshold it changes behavior qualitatively.

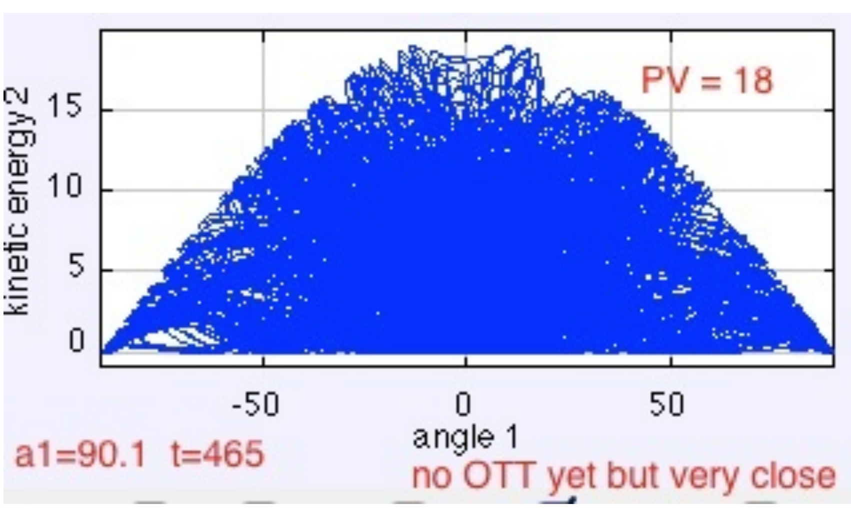
This screenshot is from a 89.9-degree run which didn't have quite enough energy to send the blue bob over the top:



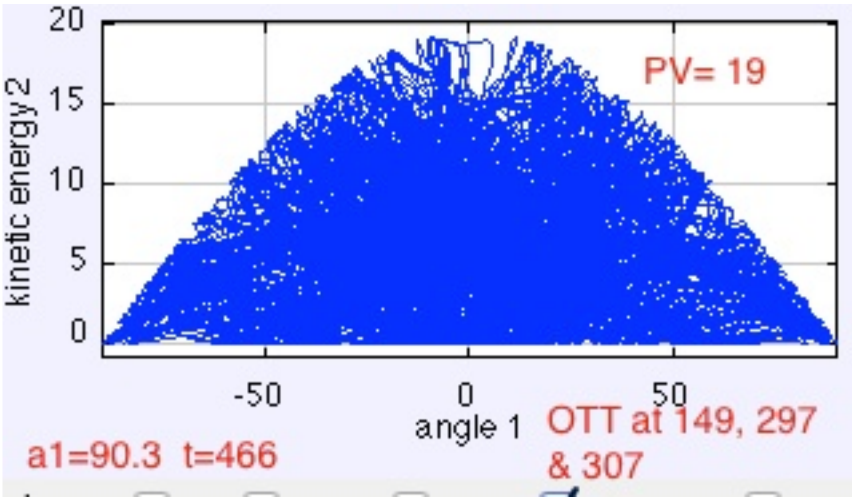

This screenshot was from a 90.1-degree run that had slightly more energy than needed to send blue over the top:




However as seen below it hadn't done so after a fairly long run lasting 486 seconds. The system must –to use a technical term- gyrate around until it reaches the magic combination of arm positions and speeds needed to transfer over 99 percent of total system energy into blues PE so as to lift it high enough.  The remainder gives it enough speed to go over.

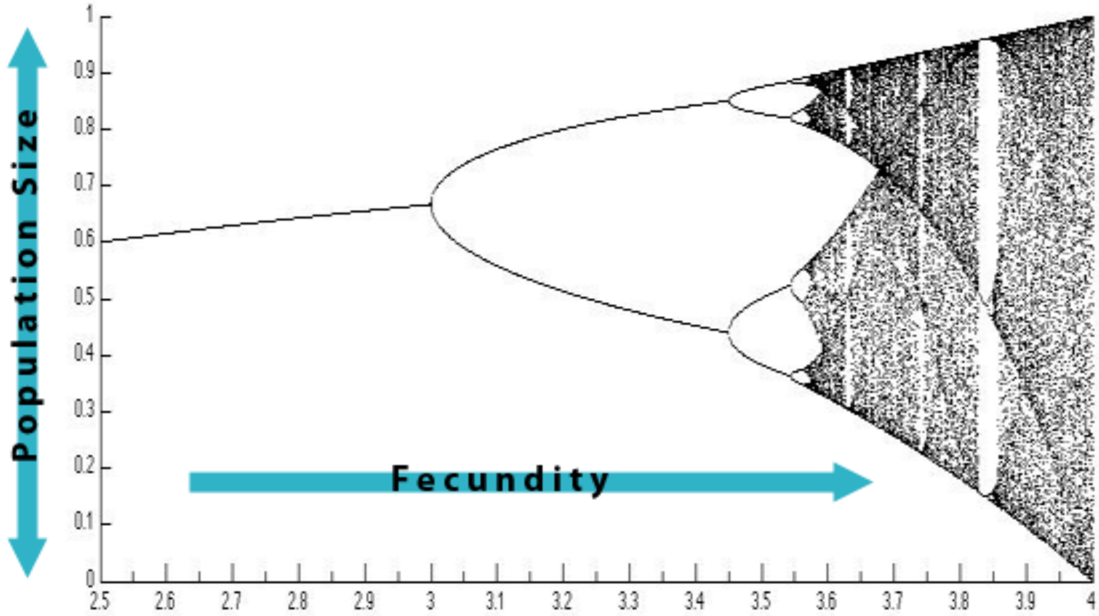


Raising the energy just a bit more caused it to go OTT three times in the following run.




As noted elsewhere if an important real-world system went over a threshold that changed its behavior in similar fashion it might have important consequences. This seems potentially relevant as global energy rises due to global warming. Might there be some natural system that crosses this threshold and begins behaving in a detrimental way? I haven't looked very hard for a potential example but this aspect of systems behavior seems worth further investigation.

Experts have also found that the threshold to chaos is sharp. The iconic bifurcation diagram below applies to the population or logistics equation. The lines at left show that system oscillating perfectly periodically whereas the shaded areas at right indicate chaos. When the reproduction rate in this equation reaches about 3.6 the system suddenly becomes chaotic. The complexity of this diagram shows that systems behavior can be quite complex because a series of "period doublings" precede chaos and chaos is interrupted by windows where the system becomes periodic again. I revert to the simple point, namely that the threshold to chaos is sharp and occurs when some parameter, which I believe generally reflects energy level, is raised above that threshold.




The image below shows how a technical measure called the Lyapunov exponent varies as the release angle (i.e.: energy level) of the double pendulum is raised. The system suddenly becomes chaotic when the line goes vertical at about 0.7 in this diagram from: http://psi.nbi.dk/@psi/wiki/The%20Double%20Pendulum/files/projekt_2013-14_RON_EH_BTN.pdf Most experts compute the Lyapunov exponent to determine whether a system is chaotic or not. I didn't have that ability using the Dooling model so needed to test for SDIC instead.

R. Ø. Nielsen, E. Have, B. T. Nielsen The Double Pendulum

The Niels Bohr Institute
March 21, 2013

http://psi.nbi.dk/@psi/wiki/The%20Double%20Pendulum/files/projekt_2013-14_RON_EH_BTN.pdf

NOTE: the system they made and modeled had solid arms not bobs. This may effect Lyap… exponent.

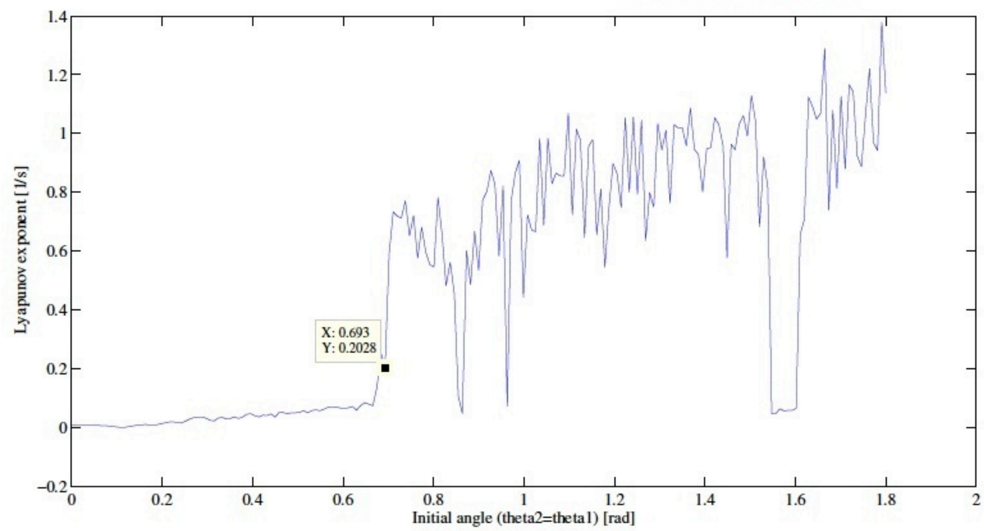


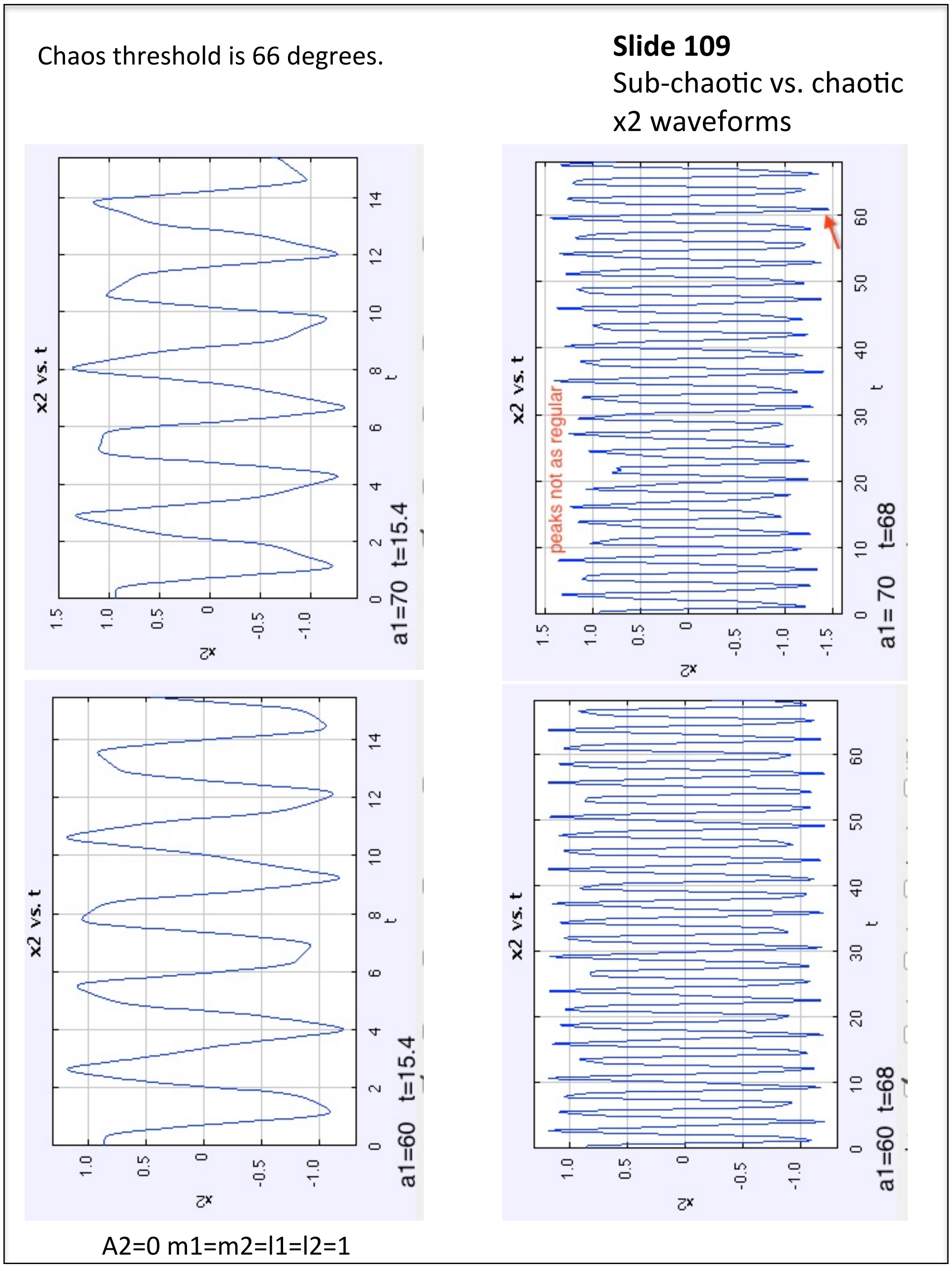

Figure 9: The Lyapunov exponent as a function of initial angle ($\theta_1(0) = \theta_2(0)$, $\dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$)

This is sufficient support for saying that the threshold to chaos is sharp.

## 5.3 Consequence of a system becoming chaotic

When increasing energy causes the double pendulum to cross a threshold and become chaotic nothing dramatic happens to the way it oscillates. This seems counterintuitive. One suspects something dramatic would happen. To investigate this I made some runs just above and below the threshold and compared the waveforms. The results are presented below.

Slide 109 compares waveforms of the double pendulum when it was sub-chaotic and quasi-periodic in a run with a1 set to 60 degrees to a chaotic run made with a1 at 70 degrees. The short-term view showing just a few cycles shows the behavior is virtually identical. The system oscillates at the same frequency, the waves are not sharper or more rounded. Their heights vary about the same amount from one cycle to the next. This suggests, not proves, that the transition into chaos would hardly be noticed in the short term by anyone depending on this system.

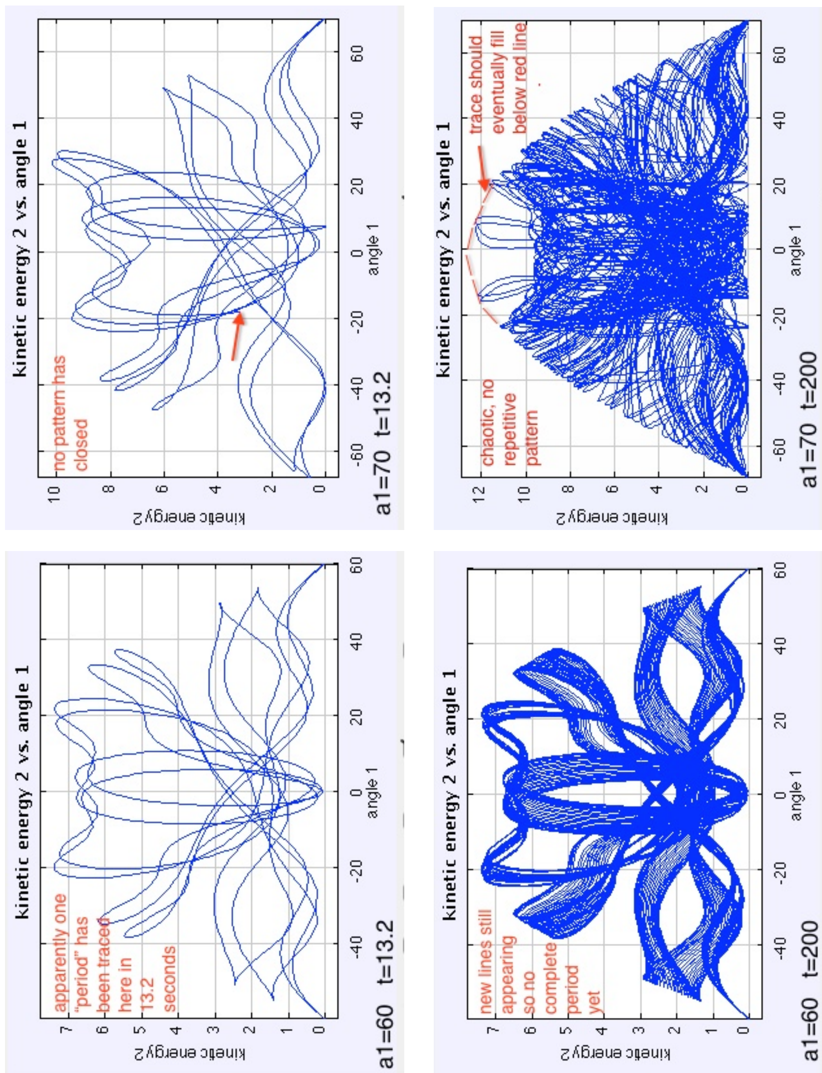



At first glance the longer-term behavior doesn't seem radically different either. The peaks and valleys grow a in magnitude but that's because the level of energy has changed. The frequency remains the same. The heights of the peaks seem to vary about the same percent. However close inspection shows that the 60-degree (quasi-periodic) run has a repetitive pattern which produces two high peaks separated by two lower ones, then two highs separated by one low and so forth. In contrast the 70 degree run is much less consistent because it's chaotic.

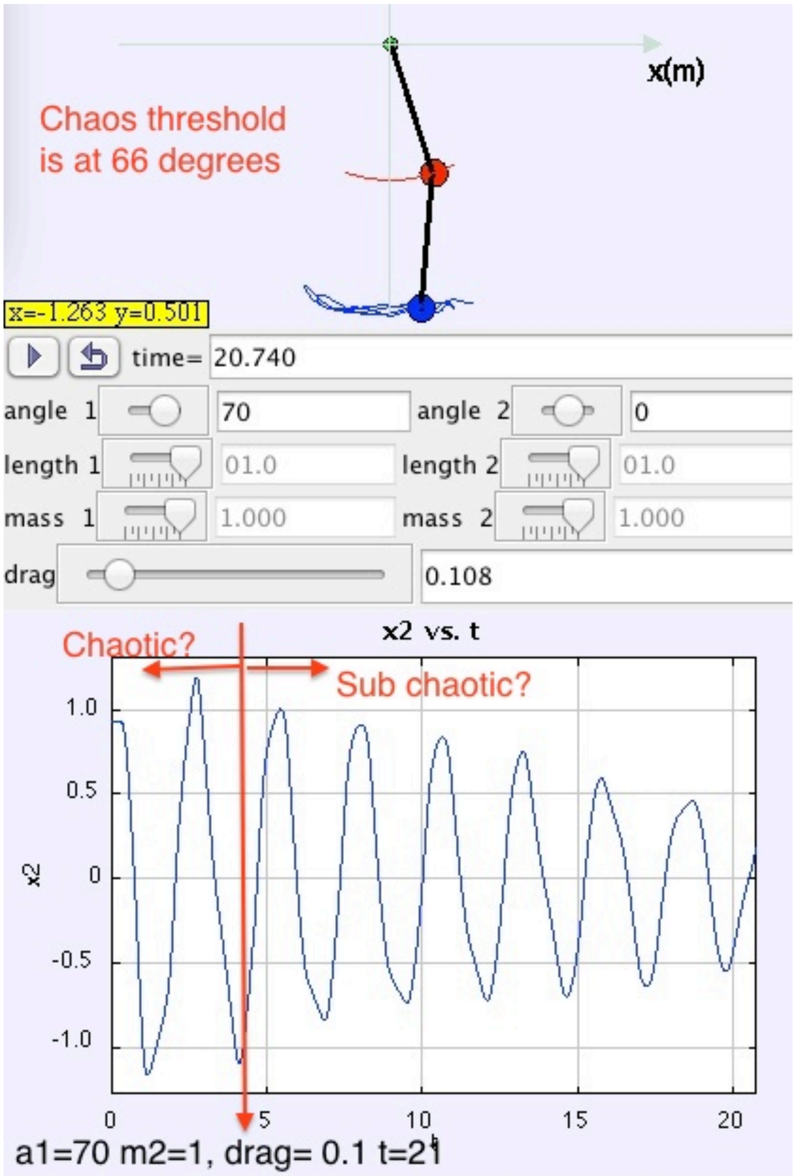
Slide 110 conveys much the same overall message. The short-term pattern of behavior is much the same. In the long term the values change over a wider range.

Chaos threshold is 66 degrees.

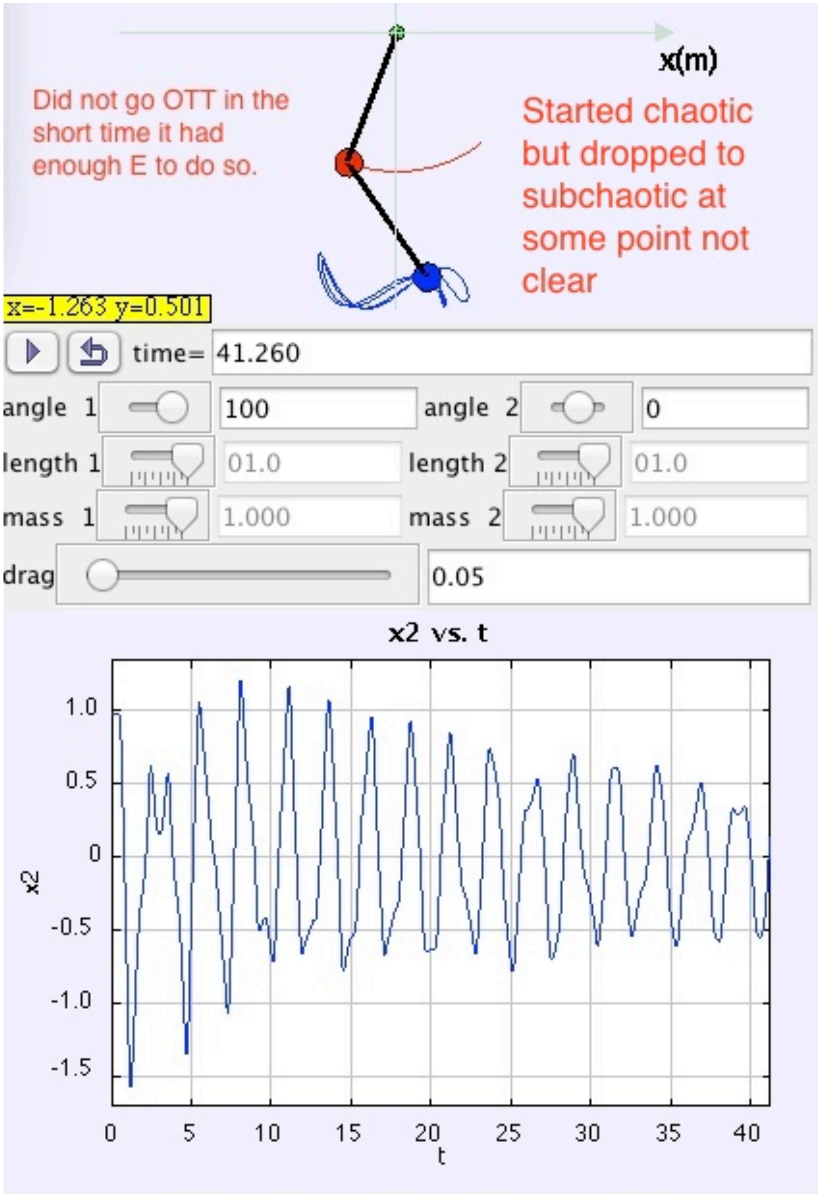
**Slide 110**
Sub-chaotic vs. chaotic
ke2/a1 patterns




Next it was decided to try simulating a system transitioning thru the chaos barrier during a single run. The model didn't support the gradual addition of energy but it did support its gradual diminution by allowing frictional drag to be included. It was assumed that the transition from sub-chaotic to chaotic would be a mirror image of the transition from chaos to sub-chaos. We know that the system was chaotic when started at 70 degrees and since the overall wave height dropped to less than it was in the 60 run it should have been sub-chaotic at the end. Just were the transition occurred is uncertain. I simply marked my best guess. IF this represents reality then nothing dramatic would happen if the double pendulum had enough energy added to make it chaotic. Viewing the real pendulum gives further support to this finding. Its movements change in a smooth, seamless manner as it slows from wildly chaotic to a stop.




The following run started from a still higher energy. It transitioned the threshold without any noticeable impact on the waveform.

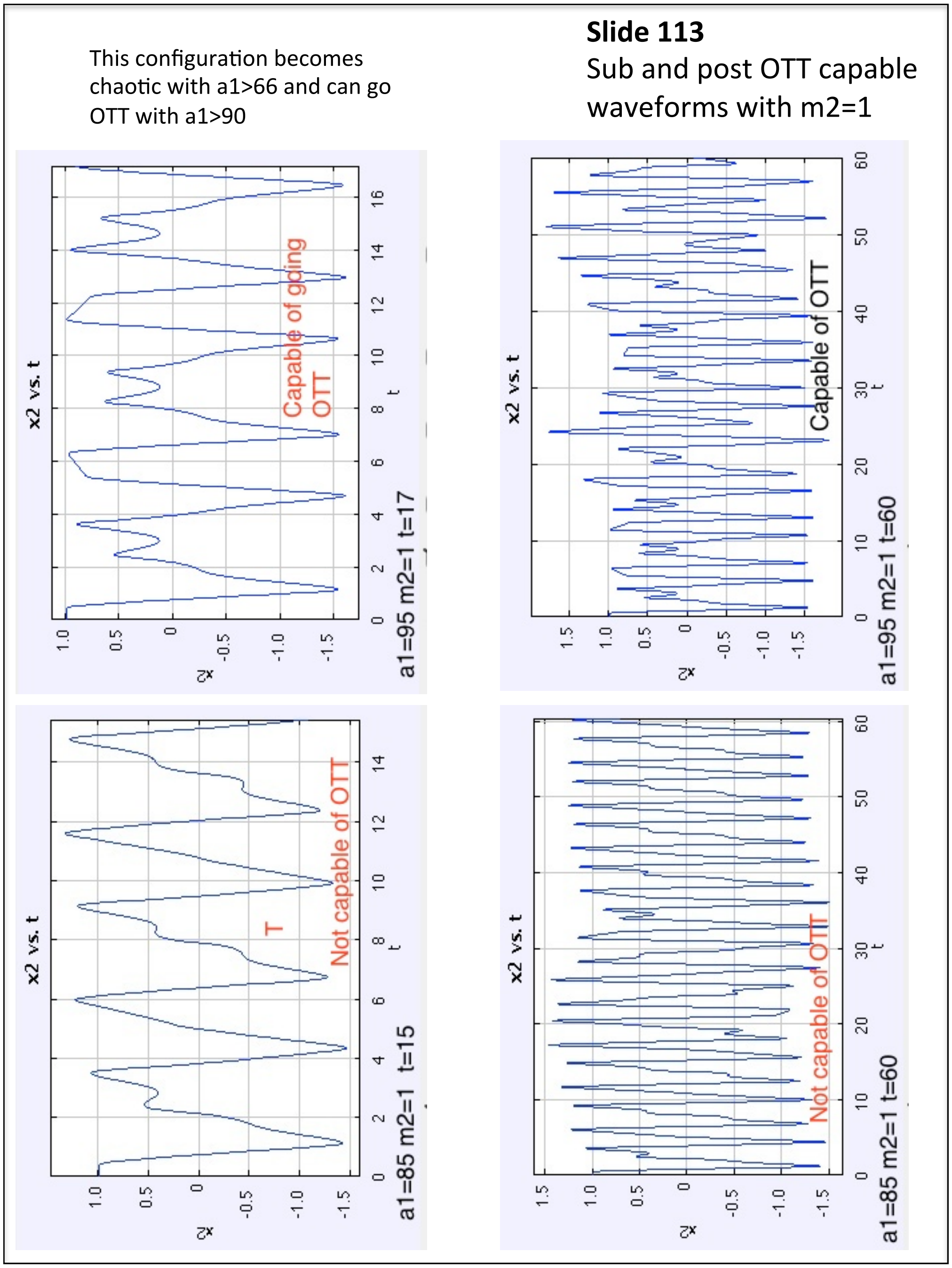



**IF these results apply to real-world systems it suggests there would be no drastic and immediate change in behavior if they were driven into chaos by the addition of energy.** I cannot reconcile this opinion with the drastic change that occurs when a regularly beating heart becomes arrhythmic. Perhaps that's an entirely different situation.

## 5.4 Consequence of qualitative changes in behavior

The only qualitative change in behavior investigated in this book was when the outer bob in the double pendulum swung over the top occasionally as opposed to always reversing and falling back. This is the most dramatic thing one notices when watching the double pendulum so I often call going over the top (or OTT) a dramatic event. More properly it's a qualitative change in behavior. Aside from the visual impact what else does that mean? The first finding is that it brought no dramatic changes to the waveforms.

Slide 113 shows the results of examining the waveforms above and below the energy level needed to cause occasional OTT events, as was also done in section 8.2. Again no significant differences were observed.




Its hard to imagine the equivalent of the qualitative change in behavior or dramatic event described above in an important real-world system. Again I think it deserves more attention than I've been able to give it. What first occurred was to wonder if some ocean current might suddenly change direction as a result of increased energy due to global warming. I briefly considered the el Nino current but at first glance it didn't seem a likely candidate, partly because it seemed capable only of reversing a bit but never circulating in a full loop.

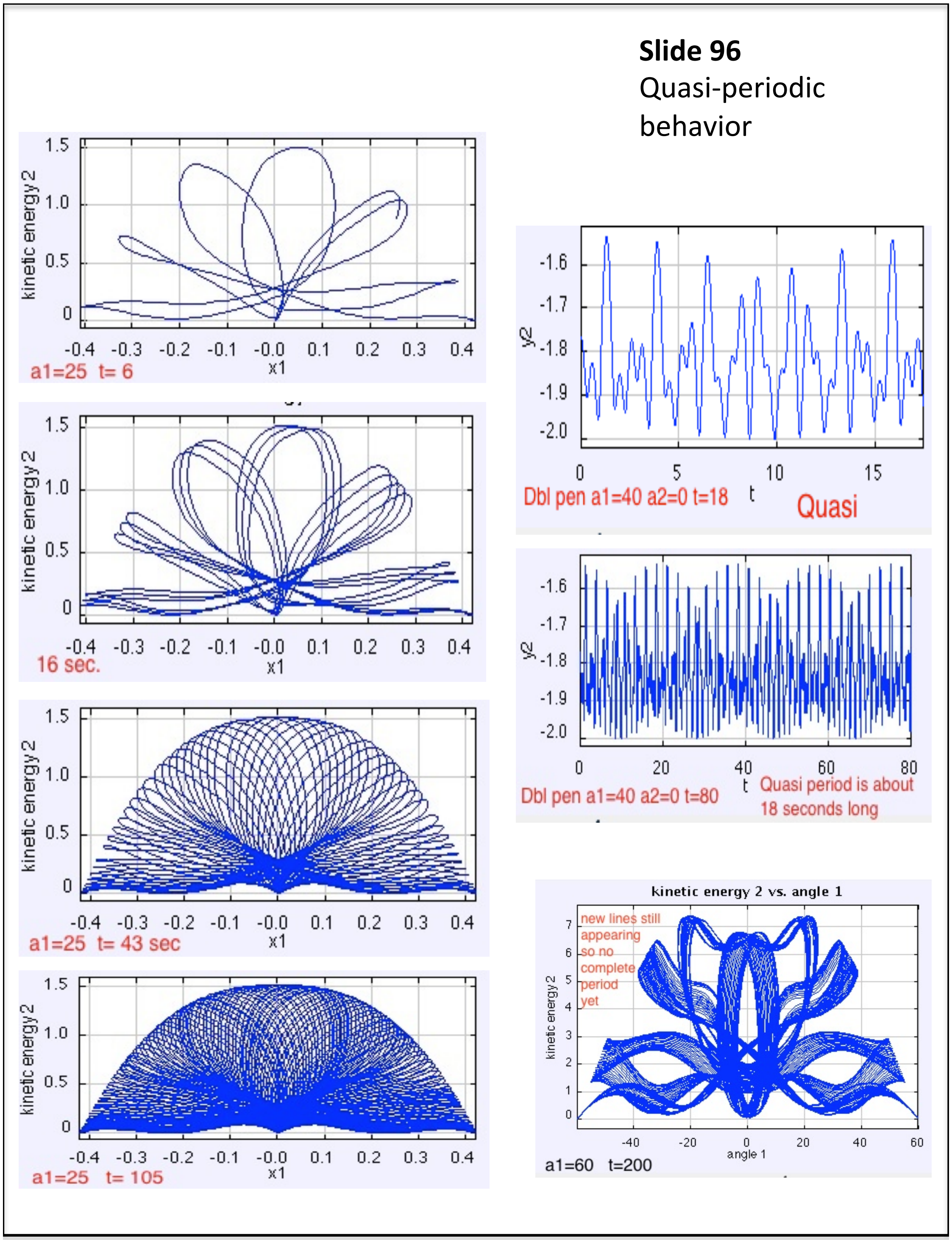
So I can only leave this question. This double pendulum experienced qualitative changes in behavior. Does such behavior apply to real-world systems like ocean currents or ecological systems?

Finally, note that it might not make any practical difference to the system, like an ocean current if it does something dramatic like reversing. What might be important is some other system or situation that rides atop that system and is heavily effected by it. For instance if the Gulf Stream were to behave differently it would greatly effect the climate of northern Europe. I suspect the Gulf Stream could be affected by global temperature increases. Its apparently also affected by fresh water run-off from Greenland.

## 5.5 Prediction is relatively easy in perfectly periodic and quasi periodic systems

**Perfectly periodic:** If a system is isolated from outside interference and perfectly periodic then predicting the exact future value of an oscillating variable is simple if one has accumulated a historical record of its past behavior and know where you are on the waveform. Its obviously equally simple if one has an accurate model. The best example is our ability to predict things like sunrise and sunset times years in advance.

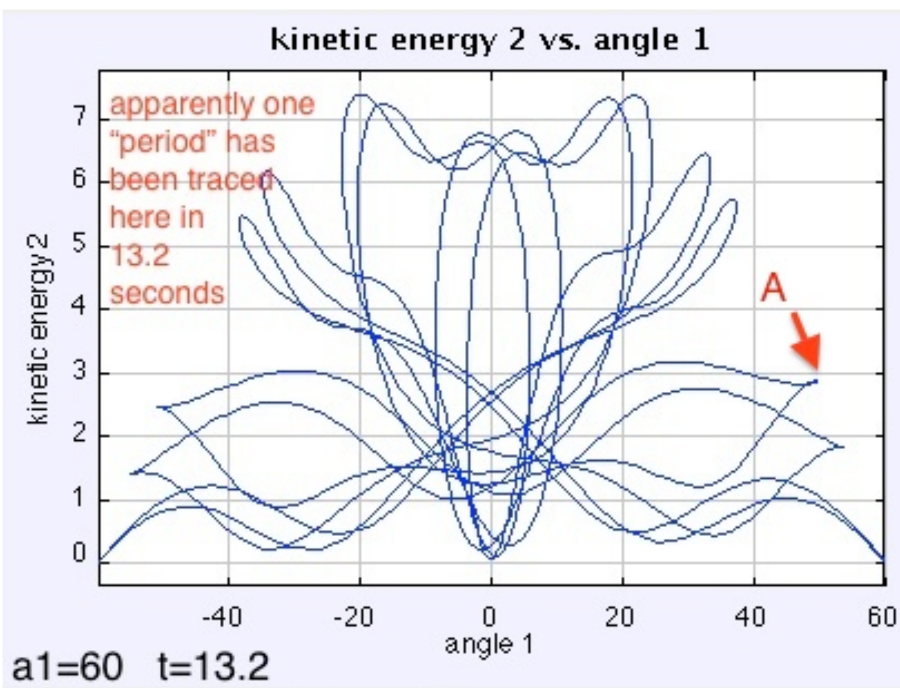
**Quasi-periodic:** A quasi-periodic system will repeat the same general waveform or the same general phase-space plot time after time. It is therefore safe to say it will continue doing so in future absent any outside disturbances. Slide 96 illustrates this.

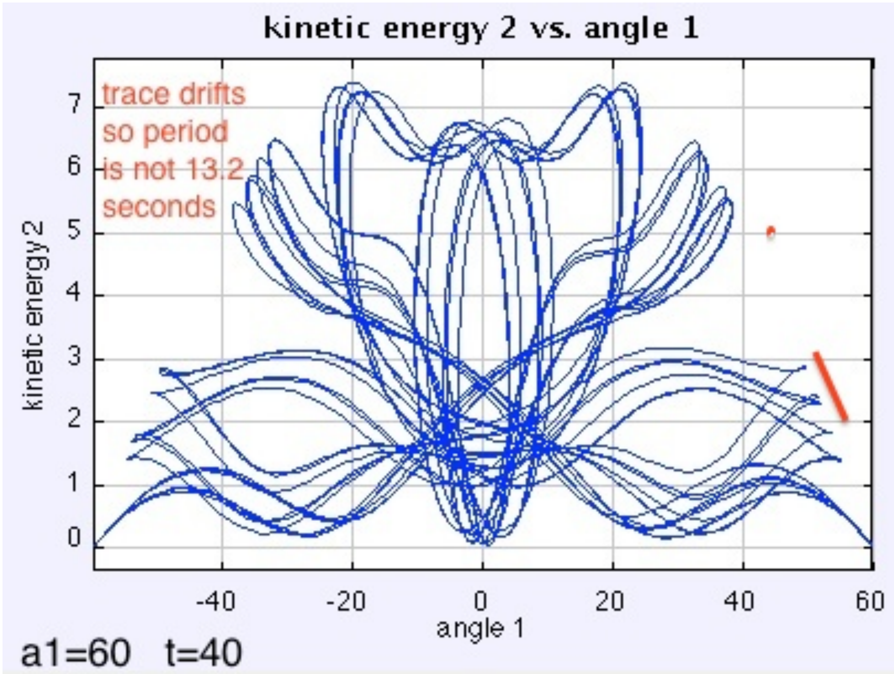



First note that the waveform at upper right is symmetrical about the valley at t=10. If one has observed it over time its easy to use a waveform like this to predict what it will do in future in terms of the general pattern of ups and downs or what the waveform in a given cycle will look like. For instance we know the shape of the waveform between t=10 and 12 will repeat every 18 seconds since that's the period in this particular example. And the longer term oscillations seen in the plot just below will also repeat every 18 seconds.

The four screenshots at left show that the general pattern of movement will also repeat but the values will drift over time so each pattern will be offset from the prior ones so the trace eventually fills the entire dome shaped envelope. I oft mention that quasi-periodic behavior is like a dance comprised of a sequence of steps that repeat over and over, except the exact length of each step varies.

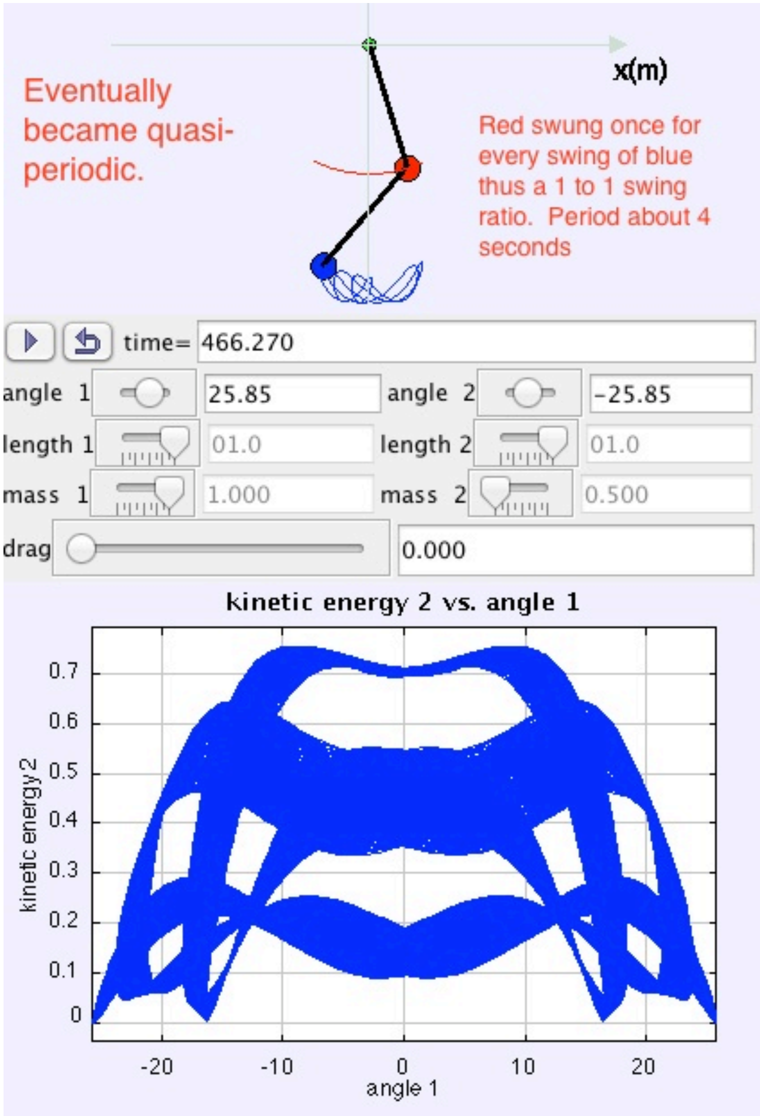
Now to the screenshots below. To further clarify suppose the pattern shown below did repeat every 13 seconds and we were measuring the variables and know we were at point A on the pattern. It would then be possible to predict approximately where we would be at any future time. For instance every 13 seconds we would be back close to A, but not precisely at A. We would be somewhere along the red line in the second screenshot.






In some runs quasi-periodic traces fill the entire envelope over time as they did above. But in other runs they remain confined in a band as shown in the screenshot below. This run lasted 466 seconds. Long enough for the pattern to repeat many times. Its not certain it would always stay within this band but in any case we can assume it would for a long time thus making it relatively easy to predict the approximate value of a variable, and of course the overall pattern of dance, over a long time. This again assumes we have observed past behavior long enough to know the system stays in a band.

I believe tidal heights are like this. They oscillate in a general manner over a lunar month, but stay within a band.




## 5.6 short-term prediction is possible in chaotic systems

Its possible to make accurate short-term predictions of the future value of some variable when a system is oscillating chaotically, providing one has a good model. This is something all experts agree on.

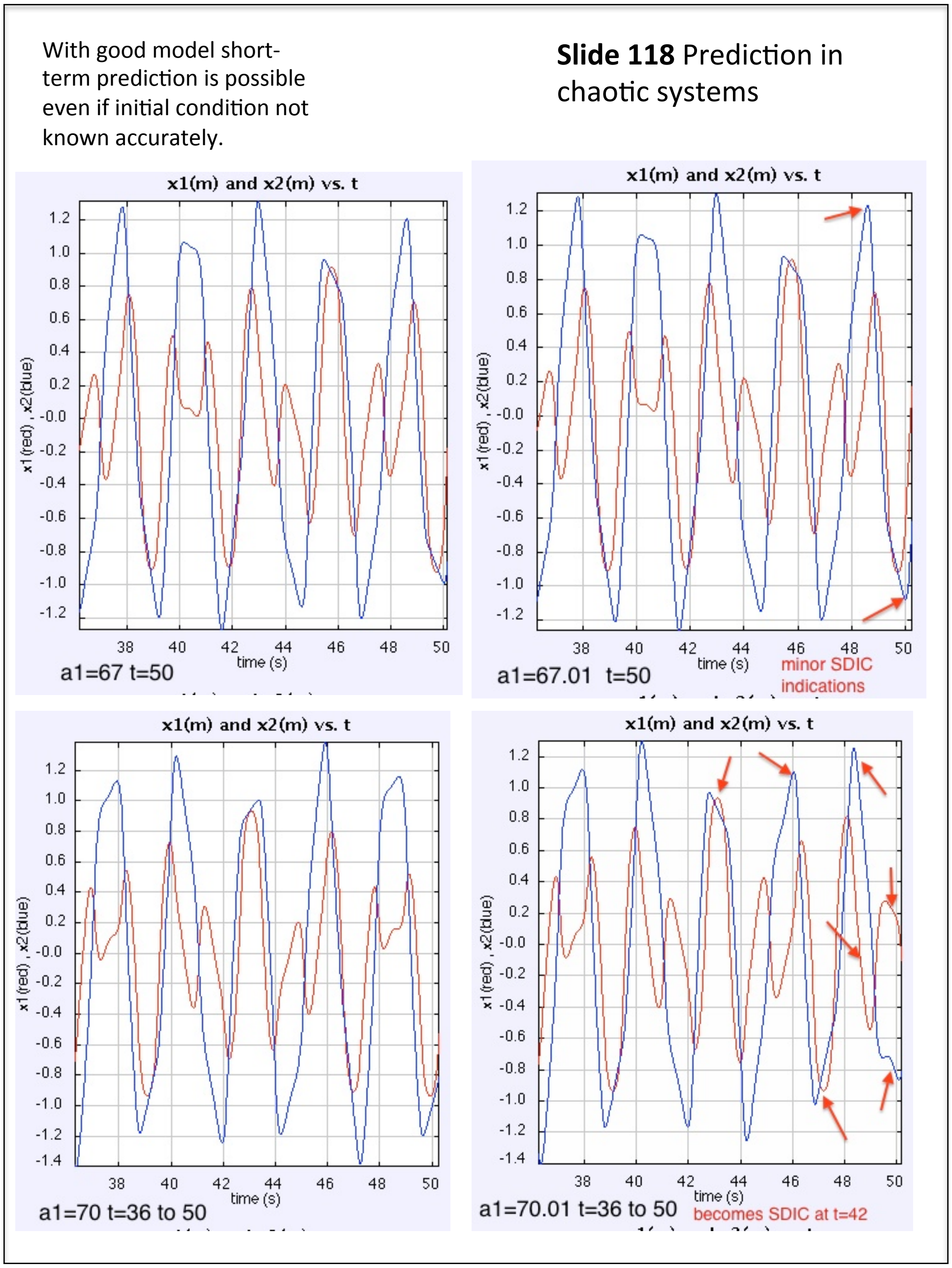
Since historical records show no periodically repetitive behavior they are useless for that purpose. However historical records can predict that the general behavior typified by random calms and spikes will continue. This is a bit trite because its equivalent to saving there have been hurricanes and floods in the past so there will be more in future.

Slide 118 shows that it is possible with a good model to make short-term predictions. The reason is that it takes time for errors in estimating initial conditions to begin affecting the accuracy of forecasts.

Consider the two top screenshots made with a1 set to about 67 degrees. Suppose our measuring instruments aren't perfectly accurate so we don't know if the true value is exactly 67 degrees or just close, like 67.01 for instance. We need to input something into the model before starting the simulation, so we probably put in 67. Careful comparison shows this uncertainty makes no difference since the waveforms are virtually identical for the first 48 seconds of this run. I other words with this model we can accurately predict the value of x1 (the blue trace) for the first 48 seconds even if we don't know the exact value of the initial condition a1 precisely. The same thing holds for the pair of runs made near 70 degrees.

Experts say this is true with weather forecasts. They can be fairly accurate a few days out.

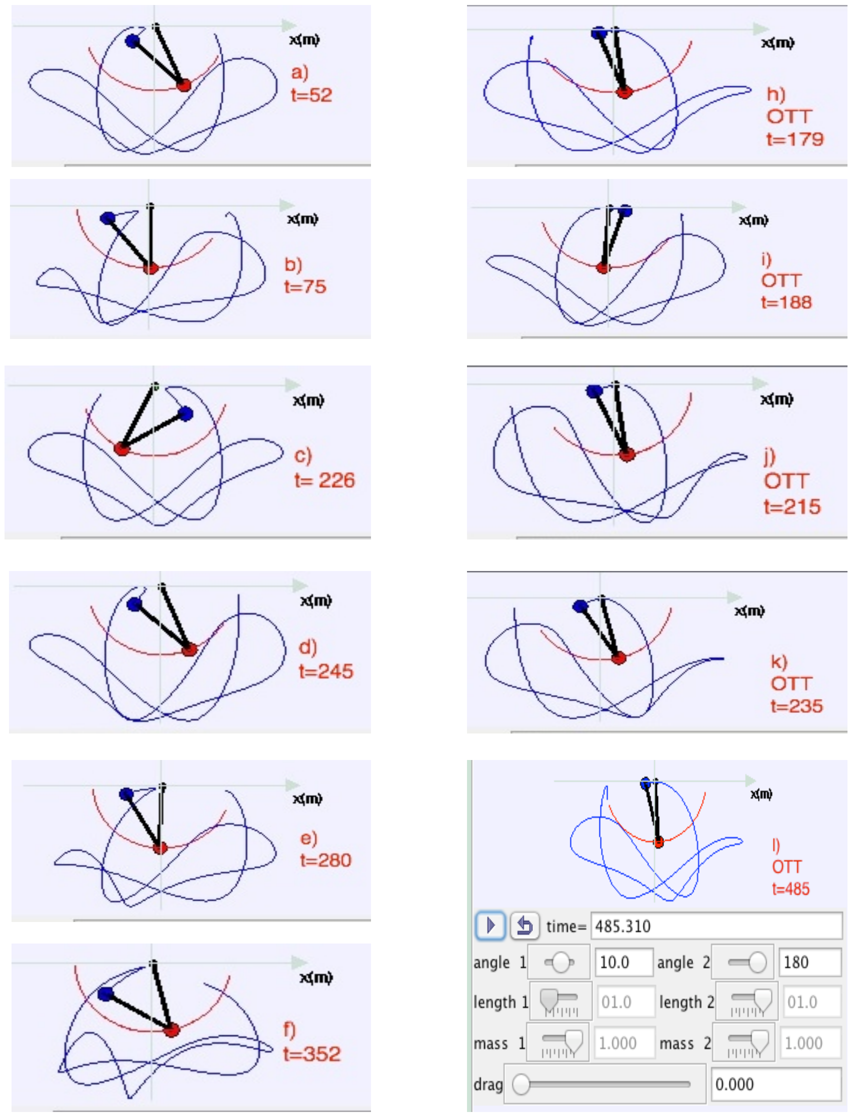
I am tempted to say that initial errors haven't had time to magnify enough to become noticeable but its not clear they actually grow from the beginning. Instead they seem to suddenly appear after a certain time has elapsed. This is treated more in Chapter 9.




**Early warning of spikes:** There appears to be a distinctive series of motions that occurs just prior to the outer bob of the double pendulum getting very near the top or going over it. (The PE of the bob spikes at that point.) If so, looking to see if such motions have started to develop might provide some short-term warning that this event was imminent. The height of the bob and thus its PE spikes during that event so this suggests a potential way to predict otherwise random spikes, albeit only a short time in advance.

Slide 1 provides some support for this idea.

**Slide 1**
Pre OTT warning?




All these screenshots were taken during a single run. It was paused whenever the bob looked like it was going to go near or over the top so a screenshot showing blues recent trajectory could be captured. I refer to the blue line. During this relatively long run the blue bob almost went over the top six times but fell back. These instances are shown in the left column. The right column shows the five instances were it went over.

Careful inspection shows that the trail or trajectory was similar in every case, albeit with subtle differences.  In short there was a distinctive windup before each pitch that sent the blue bob near or over the top.

This situation was not investigated further.  It may or may not happen at different energy levels within the double pendulum.  It may or may not happen in other chaotic systems.   However if it is a generalization that most spikes or dramatic events are preceded by a distinctive windup then watching for them to start developing may provide short-term warning the dramatic event is imminent.

After watching the double pendulum extensively I could see these windups developing with little difficulty.  I suspect some type of pattern recognition software could do so as well.

When I did this run I was trying to see if the windup before a situation where the bob almost went over the top was distinctively different from situations where it actually did go over.  I couldn't tell the difference by eye but perhaps software could detect subtle differences.

## 5.7  Long-term prediction is not possible in chaotic systems

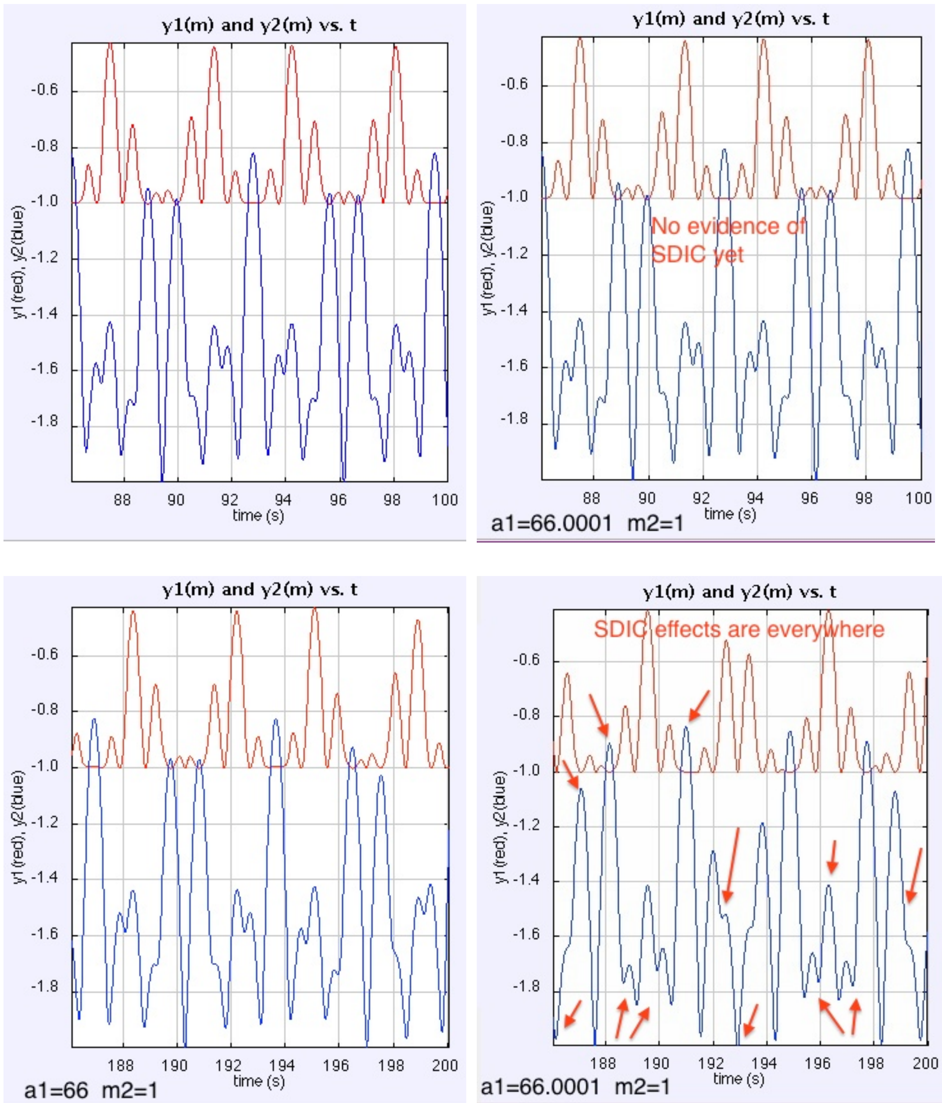
To state this more precisely we can't, even roughly, predict the value of some variable at some specific time in the future if the system is chaotic.  Experts in chaos theory all seem to agree on this point.  They make it often, and it seems to be the thing they think most important to say about chaos.

Slide 111 demonstrates that this is clearly true for the double pendulum.

With much smaller delta in initial conditions it took longer for SDIC effects to become evident.

**Slide 111**

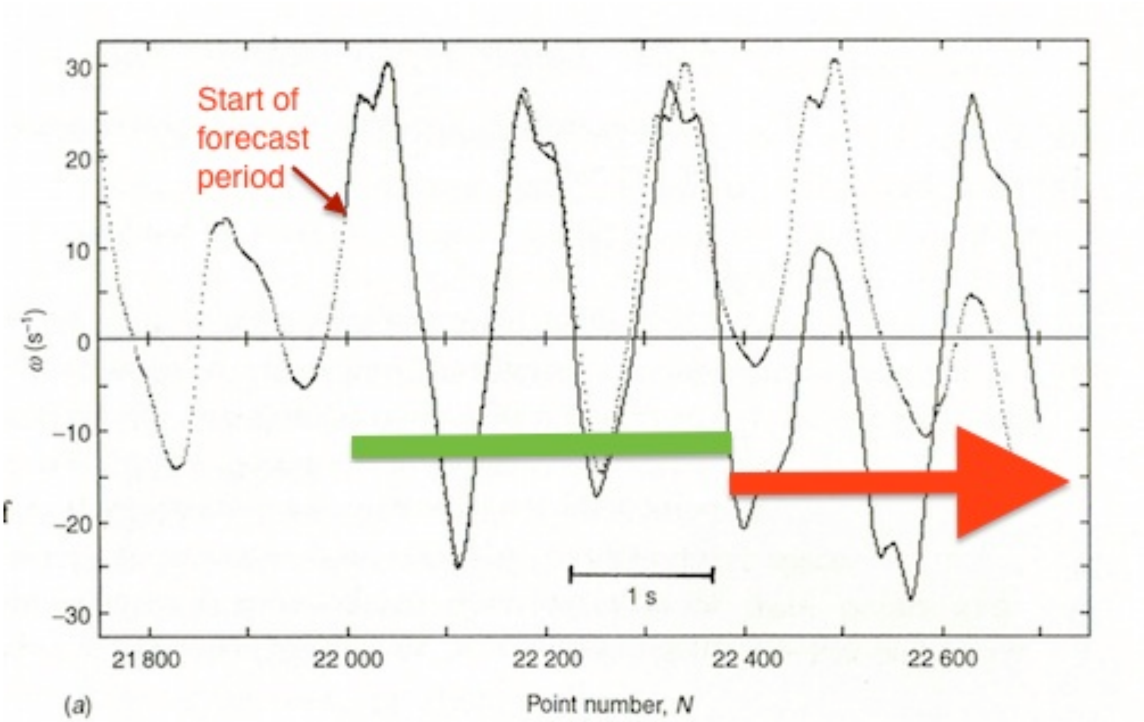
SDIC development at 66 degrees with delta a1 of only 0.0001 degrees




These runs near 66 are just over the threshold of chaos in this case. Here we tested to see if a small difference in initial conditions of only 0.0001 degrees would affect downstream waveforms, and it certainly did. The waves were virtually identical for the first 100 seconds but sometime later they began to diverge.

The lower pair of screenshots show they were very different in a sample from t=188 to 200. Take the blue waves height at any particular time and compare it between the run at 66 degrees and the run at 66.0001 degrees. I call the differences SDIC effects and marked some with red arrows. These runs are clear proof this system was "sensitive to initial conditions" or SDIC.

The image below taken from a book on chaos by Baker and Gollub shows much the same thing but perhaps more clearly. (Ca5,p.155) It apparently applies to a driven pendulum and compares the waveform from a computer forecast with the actual one from a real physical device. I added the color. It can be seen how the waveforms diverged. A reasonably accurate forecast could be made for the period highlighted in green, but the forecast didn't match reality from then on as marked by the red arrow.

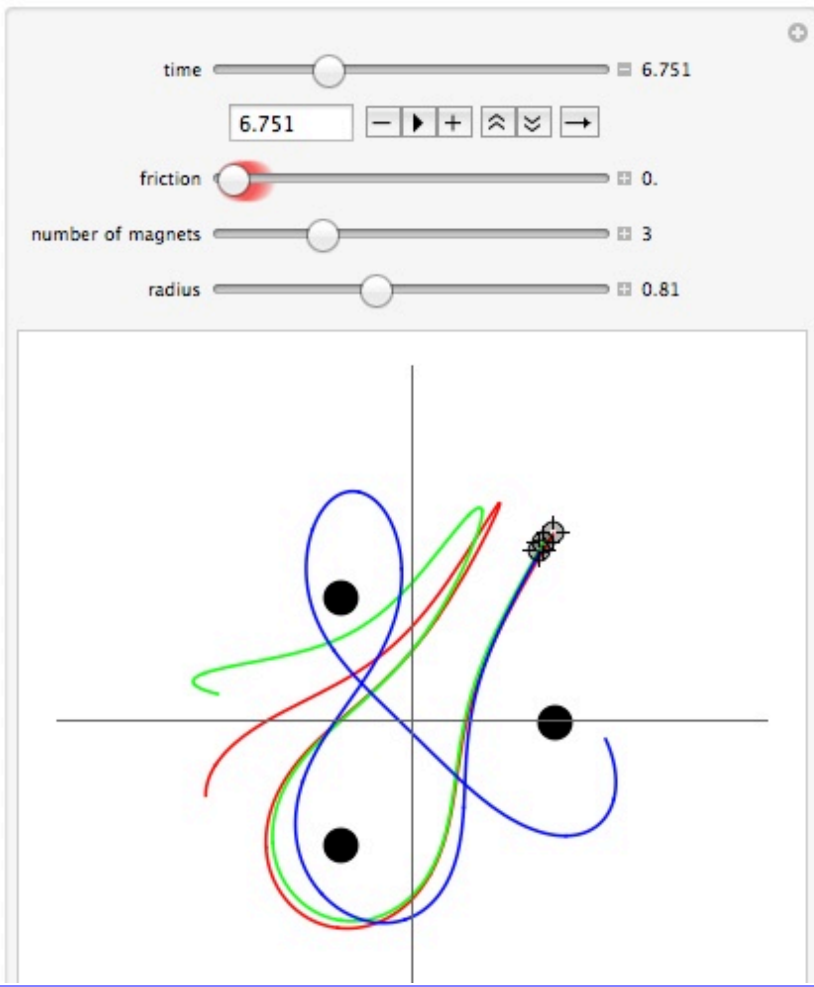



Chaotic Dynamics, by Baker and Gollub, p. 155

A video by Stogatz demonstrates the same thing more viscerally, albeit with a much larger difference in the release angel since he couldn't position them to within 0.0001 degrees of each other. Go t= 1:40 into the video at:
https://www.youtube.com/watch?v=anwl6OZ1UuQ&ebc=ANyPxKpVTVQJGdaxEk1AvyBgCXkKdrV2SKjzJxpQGeBiMpxM3F4p8oCYU4_XDCVkTD1h_nlkssunljJ-FHl56FRZdGc1wGEPWg

This video doesn't demonstrate SDIC since the two pendulums are not released together but they have very low friction and are fun to watch:
https://www.youtube.com/watch?v=N6cwXkHxLsU

The magnetic pendulum is also SDIC as shown in the screenshot below. It actually shows the paths of three different bobs computed as thought the others didn't exist. Although released from almost the same position it wasn't exactly the same position so their paths rapidly diverged. In a real world situation we wouldn't know exactly where the bob was placed before release so would be unable to predict its future location.




Note that in this run the bob did seem to start diverging from the very start in contrast to the double pendulum runs where parts only diverged visibly after some delay.

(This is just another example of why studying chaos is so difficult and why its so difficult to make generalizations that apply across all systems. In so many cases there are various differences and surprises in behavior between different examples.

Unless one explores the parametric territory very thoroughly and carefully they won't be discovered.)

## 5.8 Trying to predict the interaction of mega-trends may be useful

As explained in Chapter 11 there are many difficulties in trying to apply lessons learned about toy systems to large real-world systems. Nevertheless we must try or else all learned about toy systems has no practical value. Here and in Chapter 11. I suggest that there may be practical benefit in trying to identify mega-trends in todays world and then considering how they may interact. For instance the general increase in awareness due to TV and the internet is a world-wide mega trend in society. Global warming is a mega-trend. Globalization and automation are mega-trends. Such mega-trends may be just part of one wave or cycle in some system where mega-parts interact, but that's not certain. If true much work is needed to identify those mega-parts and how they interact. We would need to model them as a dynamic system and see if and how they oscillate. But ignoring that challenge for the moment I suggest it may be possible to create plausible scenarios that describe the combined result of several mega-trends acting together. Would they reinforce each other in some way? Would combining them, as society actually will, result in something surprising? I feel such a scenario creation effort would be worthwhile and potentially very important. I feel its one of the more tangible and potentially valuable ideas I've gotten during this research, even though it's a bit different from the most of this.

## 5.9 Can chaos be controlled?

This question has received a certain amount of expert research. The general idea has been to take advantage of SDIC so small changes made now to a system can magnify and have large, and presumably beneficial, effects downstream. For instance could some modest economic policy or intervention made now prevent a future depression. I've not had time to research this area but will offer a couple comments.

First, the system must be chaotic, and we must have a good simulation model to test potential remedies.

Second, since SDIC does amplify small perturbances there is a good chance that some other unanticipated trend or event coming from outside will cancel out the well-intended intervention or even make the situation worse. Chapter 8 discusses how no system exists in isolation so outside disturbances are a certainty and even the weakest will be magnified, per the classic butterfly effecting the weather story. The as we get closer to some undesirable situation like a depression it takes more and more power (or money) to prevent it. The same would apply in the double pendulum, If we wanted the arms to be straight down at time X a small intervention made well in advance could make that happen do to the magnifying power of SDIC. On the other hand if we delay and they're not in the right position shortly before time X it would take a strong force to move them there.

## 5.10 Why study chaos?

After trying to understand chaos so long its necessary to step back and ask if anything <u>new and practical</u> has been learned that would allow us to better understand, manage or cope with important real-world systems like environment or government. Here are some thoughts.

From an observation of past behavior we already know what real-world systems, like weather and economy, are acting chaotically and that we should be prepared for random spikes or calms in their behavior. And we know that short-term prediction is possible in some cases but long-term prediction isn't. But we, and experts in the relevant disciplines, already know that so nothing new is added except perhaps for a better theoretical understanding of why; namely that n-body systems and fluids can oscillate chaotically.

The notion that SDIC might allow us to prevent future disasters by making small interventions well in advance, while theoretically true, is useless because unknown outside disturbances would interfere. It's nice to save money by knowing in advance that such interactions might not work, but it's a negative conclusion saying more about what we can't do than what we can do.

It does not seem worthwhile to determine if some slowly oscillating real-world system now operating quasi-periodically might suddenly cross the threshold and become chaotic because its short-term behavior apparently wouldn't change much. At least that's what the data above suggests. Slowly oscillating systems like for instance climate change so slowly that we will live through only a fraction of one cycle of oscillation. Thus what happens after many cycles is of little concern.

The knowledge that no system exists in isolation greatly complicates any attempt to apply lessons about isolated toy systems toward understanding large complex real world systems.

Does it make any practical difference whether a real world system is PP quasi or chaotic? Maybe not. We already know by observation which real-world systems appear to be chaotic and experts in those areas have already tried with various degrees to success to predict their behavior. Knowing that a slowly oscillating (one cycle takes years or decades) system is PP or quasi would not be important if its near the the threshold and could soon go over because the waveform does not seem to change much. It just continues to oscillate but now its SDIC. It might be important in rapidly oscillating systems because it would bring those random calms and spikes. It may matter if a system is near the threshold of experiencing what I've called dramatic events or qualitative changes in behavior since that might immediately impact not only the system itself but also anything dependent on it.

## 5.11 Its difficult to relate toy system findings to real-world systems

Experts in many fields have become aware of chaos theory and sought to see if and how it might apply in systems in their disciplines. I've only had time to look briefly at a few of these and mention some in Chapter 12.

**However** Its difficult to relate toy system findings to real-world systems for a variety of reasons. These are the ones I've discovered, although others have undoubted discovered them also. They are discussed in more detail in Chapter 11. but here are the main points.

> 1) Its difficult to identify in real world systems "parts" that are equivalent to the discrete physical parts in most of these toy systems.
>
> 2) It difficult to identify for real world systems an equivalent to the potential and kinetic energy that exists in toy systems and determines their behavior. Is money the equivalent of energy in economic systems?
>
> 3) It difficult to identify in real world system equivalents for the physical forces between the parts. In toy systems the forces are gravity or

electromagnetic. What are their equivalents in economic, political, ecological, and social systems?

4) Oscillation only occurs because parts have mass and thus inertia. What is the equivalent of mass in the above mentioned systems? (It obviously exists because real world systems always seem to react slowly to applied forces.)

5) No real world system exists in isolation immune from the influence of other parts and systems in the overall environment.

6) Many real world systems are not comprised of a relative few discrete parts like the toy systems analyzed in this book but rather have so many parts that they may behave more like fluids and gases.

As we look at progress in linking toy systems to real world behavior the general questions would seem to come in this order:

1) Is some variable oscillating chaotically?
2) Is this waveform the result of an internally oscillating system or the result of random and possibly one-time disturbances from outside?
3) Can we identify the interacting parts, forces, energies, links?
4) Can we build a simulation model that at least roughly replicates the systems behavior?
5) Can extreme events be predicted well enough in advance so we can better prepare for them?
6) Is something happening (like global warming) that might change the system's behavior in a significant way? Would it be a sudden or gradual change? Can that something be stopped?
7) Can any deleterious behavior that is forecast with the model be prevented?

Unless we can answer "yes" to the last three questions nothing practical has been learned.

**A bridging model is needed:** As mentioned elsewhere small toy systems with just a few parts are much different than real-world systems with many parts. Thus we need to study systems with at least 6 parts to see if lessons learned from the double and magnetic pendulum and the Logistics equations apply to larger systems. Then we should look at systems with say 20 parts. I have described the a model to facilitate such experiments in Chapter 11.

## 5.12 Definition of, criteria for, and root cause of chaos are not sufficiently understood

**Definition:** According to Strogatz there is no universally accepted definition of chaos. However he offers the following.

> "Chaos is aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions" (Ca2, p.323)

Here is what Wikipedia says by way of definition:

> "Although no universally accepted mathematical definition of chaos exists, a commonly used definition originally formulated by Robert L. Devaney says that, for a dynamical system to be classified as chaotic, it must have these properties:[12]
>
> 1. it must be sensitive to initial conditions
> 2. it must be topologically mixing
> 3. it must have dense periodic orbits
>
> In some cases, the last two properties in the above have been shown to actually imply sensitivity to initial conditions.[13][14] In these cases, while it is often the most practically significant property, "sensitivity to initial conditions" need not be stated in the definition." https://en.wikipedia.org/wiki/Chaos_theory

Here's another from Math Insight:

> "A dynamical system exhibits chaos if it has solutions that appear to be quite random and the solutions exhibit sensitive dependence on initial conditions." http://mathinsight.org/definition/chaos

One of the best plain English descriptions of chaos I've found. Includes discussion of its definition: http://plato.stanford.edu/entries/chaos/#DefCha

NOTE: These all describe the symptoms of chaos. None allude to the root cause. In my view a better definition would say something like: "a randomly changing oscillation with SDIC that occurs because of X, Y and Z. "

**Criteria for:** Chaos is a very complex topic so its with trepidation I offer the following comment. I've not seen a nice list of criteria for what makes a system chaotic or makes it capable of becoming chaotic if the energy is raised enough. N-body theory suggests to me that any system comprised of three or more parts linked with non-linear forces is always chaotic regardless of energy level. In other words that criterion alone is sufficient. On the other hand maybe such systems are only chaotic at and above certain energy levels. What I've read about the three-body problem suggests the former. My work with the double pendulum favors the latter.

**Root cause:** Nowhere in all the technical literature I've read have I ever seen an explanation for the root cause of chaos. I think it's actually fairly simple, but so subtle no-ones discovered it. I say that based on my study of the double pendulum. It all must come down to how the two arms physically interact. We can measure all the variables and how they interact so it seems we should be able to identify what changes when it becomes chaotic. And do so in a qualitative intuitive manner without resort to mathematics. I admit that's just opinion.

Nevertheless in Chapter 9 I offer some ideas about how one might find the root cause of chaos in the double pendulum. It comes down to finding the root cause of SDIC by comparing waveforms and forces just above and below that threshold. Little or no math should be needed to do this. I also offer a weaker explanation for why behavior of the magnetic pendulum is, or seems to be, always aperiodic.

**Comparative mapping:** Another opinion is that more work needs to be done to fully map the behavior of a number of different toy systems and then compare them in order to get insights into root causes. By mapping I mean making runs with many different sets of initial conditions –especially different energy levels- and plotting waveforms and phase space plots. I would for instance like to turn up the energy dial in a series of double pendulum simulations and create a movie of how the phase space plot morphs. I expect it to show if a strange attractor is present and if so whether it narrows to a line when the system becomes perfectly periodic. I'd also like to see bifurcation diagrams for all the toy systems, not just the Logistics equation.

## 5.13 Other points

1) In two series of runs the double pendulum oscillated perfectly periodically just before crossing the threshold and becoming chaotic. This is probably significant and needs more study.

2) I never found period doubling in the double pendulum. It was usually quasi-periodic even at very low energy levels. Someone else should double check.

3) Since I did not conduct a full closely spaced series of runs from low to high energy there are likely other instances where the system oscillated perfectly periodically, including maybe some in the chaotic realm (as suggested by the bi-furcation diagram for the Logistics equation.)

4) All my double pendulum research was done using one simulation model.  My results should be confirmed using a different model.